

Financial Entanglement: A Theory of Incomplete Integration, Leverage, Crashes, and Contagion*

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Abstract

We propose a unified model of limited market integration, asset-price determination, leveraging, and contagion. Investors and firms are located on a circle, and access to markets involves participation costs that increase with distance. Despite the ex-ante symmetry of investors, their strategies may (endogenously) exhibit diversity, with some investors in each location following high-leverage, high-participation, and high-cost strategies and some unleveraged, low-participation, and low-cost strategies. The capital allocated to high-leverage strategies may be vulnerable even to small changes in market-access costs, which can lead to discontinuous price drops, de-leveraging, and portfolio-flow reversals. Moreover, the market is subject to contagion, in that an adverse shock to investors at a subset of locations affects prices everywhere.

Keywords: Financial frictions, Market fragmentation, Leverage, Crashes, Contagion

JEL Classification: G01, G12

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1 Introduction

There is a substantial body of empirical literature (described in detail in the next section) concluding that market integration is limited and risk sharing is imperfect. In particular, a) capital tends to stay “close” to its origin, b) reward for risk reflects — at least partially — “local” factors that one would expect to be diversifiable, and c) reductions in capital flows and the extent of market integration tend to be associated with abrupt and substantial increases in risk premia and drops in prices, along with sudden drops in leverage (“financial crises”).

We propose a tractable theoretical framework that offers a unified view of asset-price determination, endogenous market fragmentation, contagion across seemingly unrelated markets, diversity of observed investment strategies, and leverage and its implications for financial crashes.

The model features a continuum of investors and financial markets located on a circle. Investors are endowed with shares of a risky firm domiciled at their location and traded at the same location. The dividends of this security are closely correlated with dividends of securities located in nearby locations and less correlated with securities in distant locations. Even though securities in more distant locations offer greater benefits from the perspective of diversification and risk allocation, participation in such markets involves costs that grow with the distance from the current location. Such costs are reflective of the fact that informational frictions are likely to grow as investors participate in progressively more unfamiliar markets; indeed, inside our model distance on the circle should be viewed as a broad measure encompassing the magnitude of these frictions rather than a narrow measure of geographical distance. Alongside the risky markets, investors have access to a zero-net-supply bond market.

Because access to financial markets is subject to frictions, the market equilibrium features limited integration. Investors from nearby locations, who face similar cost structures, choose to participate on arcs of the circle that feature a high degree of overlap. Even though there may be many assets with no common investor, this overlap renders all markets (indirectly) “entangled.”

In such a framework, we examine four sets of issues: a) the determination of asset prices in light of limited market integration; b) the structure of optimal investment strategies, i.e., arcs of participation, leverage, and risky-asset positions chosen by investors; c) the effect of shocks to the financial technology (i.e., the magnitude of market-access costs) and their role in market crashes; and d) the propagation of “local” shocks to the financial technology. We summarize our findings about each of the four issues in turn.

a) Limited market integration implies that investors are over-exposed to the risks of locations in their vicinity. Consequently, risk premia are higher than they would be in a frictionless world. An important aspect of the analysis is that the magnitudes of risk premia and of portfolio flows are tightly linked and reflect the extent of market integration.

b) A surprising implication of the model is that, although investors in any given location are identical in every respect, their investment strategies may be diverse (that is, feature different participation arcs, risky portfolios, and, importantly, leverage). Specifically, we show that a symmetric equilibrium, i.e., an equilibrium where investors choose the same strategies may fail to exist. Instead, a fraction of investors chooses strategies featuring high leverage, high Sharpe ratios, and high participation fees, while another fraction of investors chooses unleveraged, low-Sharpe ratio, and low-fee strategies. Prices are determined so that investors are indifferent between the two types of strategies; however, the amount of capital directed to each type of investment strategy is determinate and dictated by market clearing.

The intuition for this finding is that leverage and participation decisions are complements. For a given level of risk premia, participating in more markets increases the maximum Sharpe ratio attainable, which induces investors to take more leverage. Since leverage increases the overall variance of the portfolio, it increases the marginal benefit of further market participation and diversification. This complementarity between participation and leverage decisions gives rise to multiple local extrema in the problem of determining the participation arc. It is noteworthy that these multiple local extrema arise even when the participation costs are convex.

This implication of the model is consistent with the diversity of investment strategies offered to investors by, say, mutual funds and hedge funds. More importantly, it shows that portfolio leverage may arise endogenously in fragmented markets, without requiring inherent

differences between investors.

c) An interesting situation emerges when the ability to leverage is not limitless, but rather is subject to a margin constraint.¹ In such a situation marginal variations in participation costs may cause abrupt changes in equilibrium quantities, and even a change in the type of equilibrium, accompanied by discontinuous drops in prices, leverage, and capital flows.

The intuition is as follows. An increase in participation costs makes investors reluctant to continue allocating funds to the strategies that rely more heavily on costly participation. The resulting outflows from these leveraged strategies reduce aggregate market integration and push down the prices of risky securities. The drop in prices has two effects. On the one hand, it raises the Sharpe ratio and helps restore the attractiveness of high-leverage, broad-participation strategies. On the other, via the margin constraint, it reduces the possibility of leverage and broad participation because the collateral value of risky securities declines. By constraining leverage below the optimal amount, this second effect necessitates a price drop larger than would obtain in the absence of the margin constraint. Furthermore, when the effect becomes sufficiently strong, further drops in prices can no longer attract enough capital for high-leverage, broad-participation strategies to overcome the simultaneous tightening of the leverage constraint. At this point, a marginal increase in participation costs destroys the pre-existing asymmetric equilibrium and triggers a transition to a new type of equilibrium, featuring no leveraged strategies. The consequence of this transition is that prices of risky securities, investment across locations, and the amount of leverage in the economy drop discontinuously, despite the smooth dependence of investors' objective functions and feasible choices on participation costs.

d) The fact that markets are only partially integrated may strengthen the interdependence of the prices of risky securities in different locations. We present an example where the financial technology “breaks down” (i.e., the participation costs rise to infinity) only in a subset of locations — we refer to them as the “affected” locations. We show that such an event pushes prices in almost all locations downward. This is true even for locations that are *not* connected with the affected locations through asset trade and also have negative dividend correlation with them.

¹In Appendix D we consider alternative formulations of borrowing frictions, motivated by limited liability.

The intuition for this finding is as follows. As a direct effect of the shock in the affected regions, investors in affected locations stop investing in risky markets in their vicinity. Consequently, prices in these locations fall so as to attract demand from neighboring unaffected locations. The portfolio reallocation in these neighboring locations happens away from other, farther locations, resulting in weaker demand in these farther locations, necessitating price drops in these locations as well, so as to attract demand from locations neighboring the neighboring locations, etc.

The extent of price drops required to compensate investors for tilting their portfolios to absorb local risks depends on the extent of their overall participation in risky markets. If the extent of their participation is small, so that their portfolio is heavily exposed to risks in their vicinity, then a tilt towards a nearby location requires a higher compensation. By contrast, if investors' portfolios are invested across a broad range of locations, then they are more willing to absorb risks in their vicinity. Hence, somewhat surprisingly, the higher the degree of market integration, the lower the price drop in response to a local participation shock (and vice-versa).²

In summary, the paper offers a unified framework to study several financial-market phenomena — such as leveraging, crashes, and contagion — as a result of the interplay between equilibrium prices and (endogenous) market segmentation. Besides the applications that we consider, we believe that the particularly tractable nature of the framework could be useful for studying the numerous instances in economics where frictions prevent optimal risk diversification and capital allocation across asset classes.

This article is related to several strands of theoretical and empirical literature. We discuss connections to the theoretical literature here and postpone a discussion of related empirical literature for the next section.

Through its focus on incomplete participation by all agents, our paper is related to the seminal contribution of Merton (1987) and the studies inspired by it. We endogenize the degree of participation, and choose a specific asset-universe structure. Circular structures have been repeatedly used in a variety of applications from location models in industrial

²Interestingly, albeit in a completely different setting, Allen and Gale (2000) also finds that contagion is more severe when financial interconnectivity — between deposit-taking institutions, in that model — is low.

organization (Salop (1979)) to the literature on financial networks.³ From a technical perspective, our paper proposes a novel and quite tractable structure to model risky payoffs continuously distributed on a circle. The paper also differs from the financial-network literature in setup and questions addressed. Specifically, we study a relatively standard Walrasian exchange economy with (endogenously) limited participation, while that literature typically concentrates on banking issues, network externalities, etc. Finally, our construction of an asymmetric equilibrium is based on Aumann (1966), whose analysis covers the case of non-concave investor-optimization problems in markets with a continuum of agents.

Price crashes in our model are driven by the interaction of a collateral constraint and a non-concavity in investors' optimization problems, which leads to the adoption of diverse strategies. There exists a vast literature analyzing the interaction between declining prices and tightening collateral constraints — we do not attempt to summarize this literature and simply refer to Kiyotaki and Moore (1997) for a seminal contribution. A distinguishing feature of our analysis is the following tension between two effects of price adjustments. On the one hand, as is common in papers with margin constraints, prices affect feasible leverage choices, thus restricting the set of possible trades. On the other hand, in an equilibrium featuring diverse strategies prices need to adjust to keep investors indifferent between strategies associated with different levels of leverage. This tension is most vividly illustrated in cases of discontinuous equilibrium transitions (crashes), that is, in situations in which no price adjustment can simultaneously satisfy the margin constraint and retain the attractiveness of high-leverage, high-participation strategies.⁴

Modeling price crashes through changes in the type of equilibrium is not new to the literature, which contains a number of papers featuring a combination of backward-bending demand curves and multiple equilibria (see, e.g., Gennotte and Leland (1990), Barlevy and Veronesi (2003), Yuan (2005), and Brunnermeier and Pedersen (2009)). One of our main

³An indicative and incomplete listing of papers studying networks, and in particular circular ones, in finance includes Allen and Gale (2000), Freixas et al. (2000), Allen et al. (2012), Caballero and Simsek (2012), Acemoglu et al. (2013), and Zawadowski (2013). Allen and Babus (2009) offer a survey of the literature.

⁴Krishnamurthy (2010) discusses two important aspects of a financial crisis: balance-sheet effects and the disengagement of investors from markets (specifically, due to an increase in Knightian uncertainty). Balance-sheet effects and declines in participation are also present in our framework, where the feasibility and the attractiveness of alternative leverage-participation combinations are jointly determined.

innovations relative to this literature is the endogenous coexistence of leveraged and unleveraged strategies, which owes to the complementarity we identify between participation and leverage. We also do not rely on asymmetric information and the resulting need to infer from prices, as many of these papers do.

Finally, the domino effect produced by our model relates it to the vast literature on contagion. We do not attempt to summarize this literature. Instead, we single out as a natural counterpoint a particular mechanism proposed in many models as an explanation of contagion. Specifically, many papers⁵ posit the existence of some agents, frequently of limited risk-bearing capacity, who price all the assets and whose marginal utility therefore transmits shocks from one asset to the price of another. In contrast, contagion in our model obtains even though no agent participates in all markets. A shock in one area is transmitted to a distant one by affecting first the immediately neighboring areas, which in turn affect their neighboring areas, etc. A practical implication is that there can be positive interdependence between the prices in locations with negatively correlated dividends and no common traders.

The paper is organized as follows. Section 2 presents in greater detail the empirical evidence underlying and motivating this paper. Section 3 presents the baseline model. Section 4 presents the solution and the results. Sections 5 and 6 study crashes, respectively contagion. Finally, extensions, proofs and further discussion on some of the model assumptions are contained in the Appendix.

2 Motivating Facts

As a motivation for the assumptions of the model, we summarize some well-documented facts concerning the allocation of capital. Table 1 summarizes the evidence on so-called “gravity” equations in international finance. These gravity equations typically specify a log-linear relation between bilateral flows in various forms of asset trade (equities, bonds, foreign direct investment, etc.) and the sizes of the countries and the geographical distance between them.

A striking and robust finding of this literature is that bilateral capital flows and stocks

⁵See, e.g., Kyle and Xiong (2001), Cochrane et al. (2008), or Pavlova and Rigobon (2008).

Table 1: Gravity Equations in International Trade and Finance.

The table reports a survey of the literature on gravity equations in international trade and finance. Portes and Rey (2005) uses bilateral equity flow data among 14 countries from 1989 to 1996. Buch (2005) uses the stock of assets and liabilities of banks from 1983 to 1999. Head and Ries (2008) uses bilateral FDI stocks with 30 OECD reporters and 32 partners in year 2001. Talamo (2007) uses FDI flow data from 1980-2001. Aviat and Coeurdacier (2007) uses bilateral trade data and bank asset holding data in year 2001. Ahrend and Schweltnus (2012) uses IMF’s Consolidated Portfolio Investment Survey (CPIS) in 2005-2006, which reports bilateral debt investment for 74 reporting countries and 231 partner countries. Typically, the regression performed is

$$\log(X_{i,j}) = \alpha + \beta_1 \log(GDP_i) + \beta_2 \log(GDP_j) + \beta_3 \log(Distance_{i,j}) + controls + \epsilon_{i,j}$$

where $X_{i,j}$ is the equity flow, portfolio holding, FDI flow, FDI stock, trade, or bank asset holdings.

Source	Dependent variable	Distance	t-stat
Portes and Rey (2005)	Equity flows	-0.881	-28.419
Buch (2005)	Bank asset holdings	-0.650	-12.020
Talamo (2007)	FDI flow	-0.643	-9.319
Head and Ries (2008)	FDI stock	-1.250	-17.361
Aviat and Coeurdacier (2007)	Trade	-0.750	-10.000
Aviat and Coeurdacier (2007)	Bank asset holdings	-0.756	-8.043
Ahrend and Schweltnus (2012)	Bond holdings	-0.513	-4.886

decay substantially with geographical distance. This finding is surprising, since countries that are geographically distant would seem to offer greater diversification benefits; hence one would expect distance to have the opposite sign from the one found in regressions. The literature typically interprets this surprising finding as evidentiary of informational asymmetries that increase with geographical distance — a crude proxy for familiarity and similarity in social, political, legal, cultural, and economic structures. Supportive of this interpretation is the literature that finds a similar relation between distance and portfolio allocations in domestic portfolio allocations.⁶

Further supportive evidence is provided by literature that documents how partial market

⁶For instance, Coval and Moskowitz (1999) shows that US mutual fund managers tend to overweight locally headquartered firms. Coval and Moskowitz (2001) shows that mutual fund managers earn higher abnormal returns in nearby investments, suggesting an informational advantage of local investors. Using Finnish data, Grinblatt and Keloharju (2001) shows that investors are more likely to hold and trade the stocks of Finnish firms that are located close to the investors. Similar evidence is presented in Chan et al. (2005): Using mutual fund data from 26 countries, and using distance as a proxy for familiarity, this paper finds that a version of the gravity equation holds for mutual fund holdings. That is, the bias against foreign stocks is stronger when the foreign country is more distant. In a similar vein, Huberman (2001) documents familiarity-related biases in portfolio holdings.

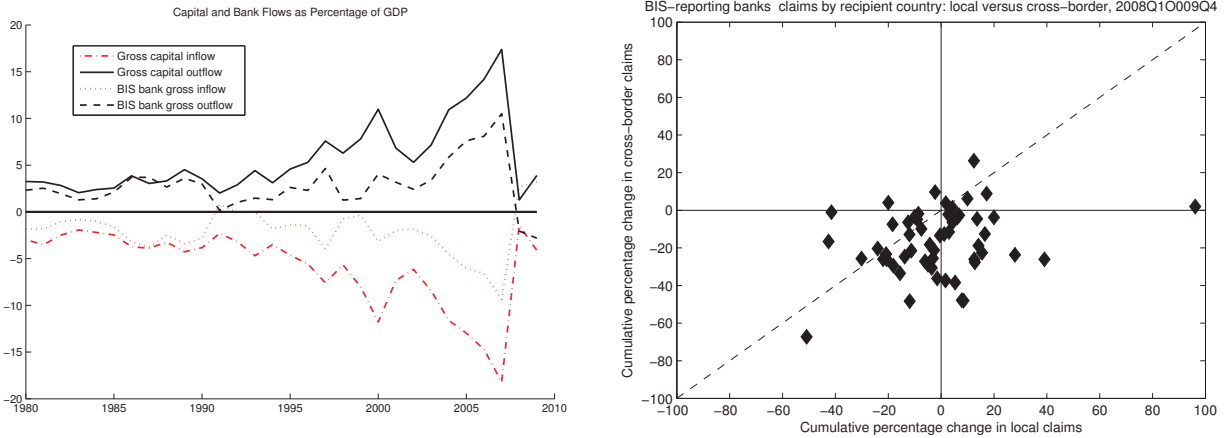


Figure 1: Reversals in global capital flows. Source: Hoggarth et al. (2010). The left plot depicts the sum of global net purchases of foreign assets by residents (labeled “Gross capital inflows”) and the sum of global net purchases of domestic assets by foreigners (labeled “Gross capital outflows”). The figure also reports cross-border bank inflows and outflows based on BIS data. Right plot: Cumulative percentage in local claims held by banks against cumulative percentage change in cross-border claims during the 2008Q1 - 2009Q4 period. Source: Hoggarth et al. (2010). based on BIS Data on the 50 largest debtor countries by foreign liabilities. For a detailed description of the sample of reporting countries, see Hoggarth et al. (2010).

integration affects the pricing of securities. Bekaert and Harvey (1995) finds that local factors affect the pricing of securities and are not driven out by global factors. An important additional finding of Bekaert and Harvey (1995) is that the relative importance of global and local factors is time varying, suggesting time-varying integration between markets.

Indeed, crisis periods offer an opportunity to visualize the extent of variation in market integration. The left plot of Figure 1 reports the sum of global net purchases of foreign assets by residents (labeled “Gross capital inflows”) and the sum of global net purchases of domestic assets by foreigners (labeled “Gross capital outflows”). The figure also reports cross-border bank inflows and outflows based on BIS data. The picture helps visualize that, in the years preceding the financial crisis of 2008, there was a large increase in gross capital flows. This expansion in capital flows came to a sudden stop in the first quarter of 2008, as the financial crisis took hold. In line with the patterns illustrated in the left plot of the figure, Ahrend and Schweltnus (2012) documents a significantly stronger coefficient on distance in cross-border gravity equations during 2008-2009. This evidence suggests that capital becomes more concentrated “locally” during times of crisis.

The right plot of Figure 1 provides a further illustration of capital concentration during the crisis by focusing on readily available BIS data. It depicts the cumulative percentage change in banks' cross-border claims against the respective cumulative change in their local claims for various countries between 2008Q1 - 2009Q4. Most points on the graph are below the 45-degree line, suggesting that the liquidation of the foreign holdings of banks was disproportionately larger than the respective liquidation of their local holdings. Direct evidence of a "flight to home" effect is provided by Giannetti and Laeven (2012), which shows that the home bias of lenders' loan origination increases by approximately 20% if the bank's home country experiences a banking crisis. Giannetti and Laeven (2012) also argues that this flight to home effect is distinct from flight to quality, since borrowers of different quality are equally affected.

We note in passing that the increased portfolio concentration during times of crisis is not limited to international data. For instance, the collapse of the (non-agency) securitization market during the financial crisis implied increased limitations to the ability of local issuers to diversify local real-estate risk.

In summary, the empirical evidence supports the following broad conclusions: a) Capital stays "close" to its origin, which implies a "local" concentration of risk in investors' portfolios. b) The extent of market integration is time-varying. Furthermore crises are times when financial integration diminishes quite abruptly.

3 Model

3.1 Investors and firms

Time is discrete and there are two dates, $t = 0$ and $t = 1$. All trading takes place at time $t = 0$, while at $t = 1$ all payments are made and contracts are settled. Investors are price takers, located at different points on a circle with circumference normalized to one. We index these locations by $i \in [0, 1)$. Investors have exponential utilities and maximize

expected utility of time-1 wealth

$$E[U(W_{1,i})] = -E[e^{-\gamma W_{1,i}}], \quad (1)$$

where $W_{1,i}$ is the time-1 wealth of an investor in location i . The assumptions that investors only care about terminal wealth and have exponential utility are made for tractability and in order to expedite the presentation of the results. In Appendix A.1 we extend the model to allow for consumption over an infinite horizon.

Besides having identical preferences, investors at any given location are also identical in terms of their endowments and their information sets. Specifically, at time $t = 0$ the investors in location i are equally endowed with the total supply of shares (normalized to one) of a competitive, representative firm, which is domiciled at the same location i . Each firm pays a stochastic dividend equal to D_i in period 1.

We specify the joint distribution of the dividends D_i for $i \in [0, 1)$ so as to obtain several properties. Specifically, we wish that 1) firms be ex-ante symmetric, that is, the marginal distribution of D_i be independent of i ; 2) the total dividends paid $\int_0^1 D_i di$ be constant (normalized to one);⁷ 3) firms with indices close to each other (in terms of their shortest distance on the circle) experience a higher dividend correlation than firms farther apart; and 4) dividends at different locations be normally distributed.

To formalize these notions, we let Z_i denote a standard Brownian motion for $i \in [0, 1]$, and we also introduce the Brownian bridge

$$B_i \equiv Z_i - iZ_1 \text{ for } i \in [0, 1].$$

We note that — by construction — the Brownian Bridge $B(i)$ satisfies $B_0 = B_1 = 0$. Using these definitions, we let D_i be defined as

$$D_i \equiv 1 + \sigma \left(B_i - \int_0^1 B_j dj \right), \quad (2)$$

⁷This assumption is made purely for ease of exposition. It is straightforward to introduce aggregate risk by adding a common, normally distributed disturbance to dividends in all locations, without affecting our results.

where $\sigma > 0$ is a constant controlling the volatility of the dividend process. Since the specification (2) plays a central role in our analysis, we show first that D_i satisfies properties 1–4 mentioned above, and then we provide a graphical illustration of these properties to build intuition.

Lemma 1 *D_i satisfies the following properties.*

1. *(Symmetry and univariate normality) The marginal distribution of D_i is the same for all i . Specifically, D_i is normally distributed with mean 1 and variance $\frac{\sigma^2}{12}$.*
2. *(No aggregate risk) $\int_0^1 D_i di = 1$.*
3. *(Continuity on the circle) Let $d(i, j) \equiv \min(|i - j|, 1 - |i - j|)$ denote a metric on the interval $[0, 1)$. Then D_i is continuous (a.s.) on $[0, 1)$ if the interval $[0, 1)$ is endowed with the metric d .*
4. *(Joint normality and distance-dependent covariance structure) For any vector of locations $i = (i_1, i_2, \dots, i_N)$ in $[0, 1)$, the dividends $(D_{i_1}, \dots, D_{i_N})$ are joint normal, with covariances given by*

$$\text{cov}(D_{i_n}, D_{i_k}) = \sigma^2 \left(\frac{1}{12} - \frac{d(i_n, i_k)(1 - d(i_n, i_k))}{2} \right). \quad (3)$$

It is easiest to understand the properties of D_i by using a graphical illustration. The left plot of Figure 2 provides an illustration of the interval $[0, 1)$ “wrapped” around as a circle with circumference one. The metric $d(i, j)$ can be thought of as the length of the shortest arc connecting i and j , as the right plot of Figure 2 illustrates. Figure 3 illustrates a path of Z_i and the associated paths of B_i and D_i .

A remarkable property of the dividend structure (2) is that the covariance, and therefore correlation, of dividends in any locations i and j depend exclusively on the distance $d(i, j)$ between the two locations, but not the locations themselves. Equation (3) gives an explicit expression for the covariance of the dividends at different locations. The associated

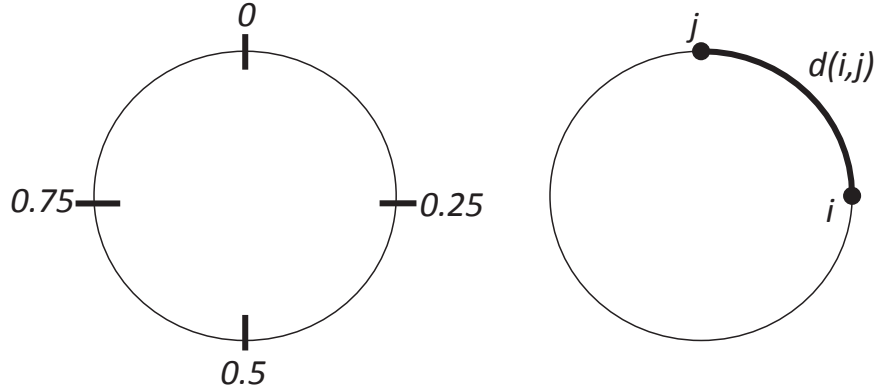


Figure 2: The left plot depicts a circle with circumference 1. The bold arc on the right plot depicts the notion of the shortest distance on the circle $d(i, j)$ between points i and j that we use throughout.

correlations follow immediately as

$$\text{corr}(D_i, D_j) = 1 - 6d(i, j)(1 - d(i, j)). \quad (4)$$

Equation (4) implies that the correlation between D_i and D_j approaches one as the distance $d(i, j)$ approaches zero, and is minimized when $d(i, j) = \frac{1}{2}$, i.e., when the two firms are located diametrically opposite each other on the circle.

3.2 Financial markets and participation costs

Investors in location i can trade claims to the stock of firms located in every location $j \in [0, 1)$ on the circle. Motivated by the evidence presented in Section 2 — specifically the fact that capital tends to stay close to its origin — we assume that participation in financial markets is costly, and the more so the farther away a financial market is from an investor’s location. Gârleanu et al. (2013) provides one potential motivation behind this assumption by introducing informational asymmetries that grow with distance, impact expected returns, and can be overcome by incurring a distance-dependent cost. We provide further details on this motivation in Appendix C, and proceed with our analysis taking the participation costs as given.

We next propose a mathematical structure for the participation costs. The participation

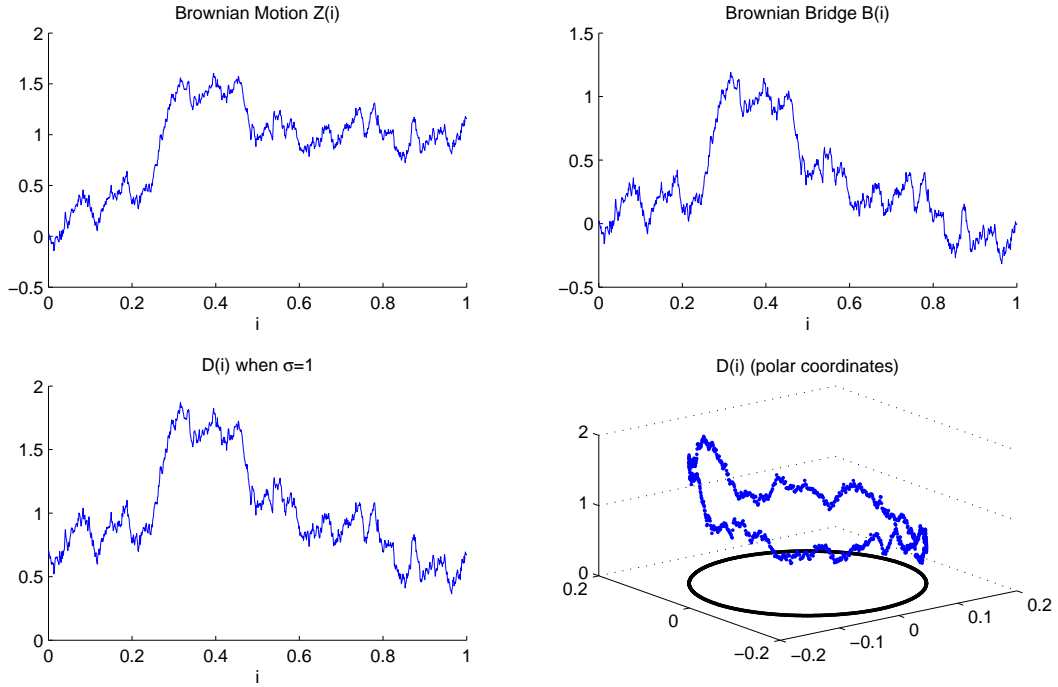


Figure 3: An illustration of the construction of the dividend process D_i . The two plots at the top depict a sample Brownian path Z_i , and the associated path of a Brownian bridge B_i . The two bottom plots depict the associated sample path of D_i , when the indices $i \in [0, 1)$ are aligned on a line and when the same interval is depicted as a circle with circumference one.

decision of investors consists of choosing a subset of all markets $[0, 1)$ in which to invest. To avoid unnecessary complications, we restrict attention to subsets of $[0, 1)$ that can be represented as a finite union of disjoint intervals with midpoints $a_{i,n}$ and lengths $\Delta_{i,n}$. Hence, the participation-cost function is a mapping from such subsets of $[0, 1)$ to the set of positive real numbers. For instance, if an investor in location i chooses to participate in N_i such intervals with midpoints $a_{i,n}$ and total length $\Delta_i \equiv \sum_{n=1}^{N_i} \Delta_{i,n}$, she incurs costs equal to

$$F_i(\{a_{i,1}, \dots, a_{i,N_i}\}, \Delta_i) = \kappa \left(b_{N_i} \sum_{n=1}^{N_i} f(d(a_{i,n}, i)) + g(\Delta_i) \right), \quad (5)$$

where $\kappa > 0$, and $f : (0, \frac{1}{2}] \rightarrow \mathbb{R}$ and $g : [0, 1) \rightarrow \mathbb{R}$ are positive, non-decreasing, differentiable, and convex functions. We assume that f has a discontinuity at zero in the sense that $\lim_{x \rightarrow 0} f(x) > 0$, while $f(0) = 0$. Also, $g(0) = 0$ and $g'(0) = 0$. Finally, b_{N_i} is positive and

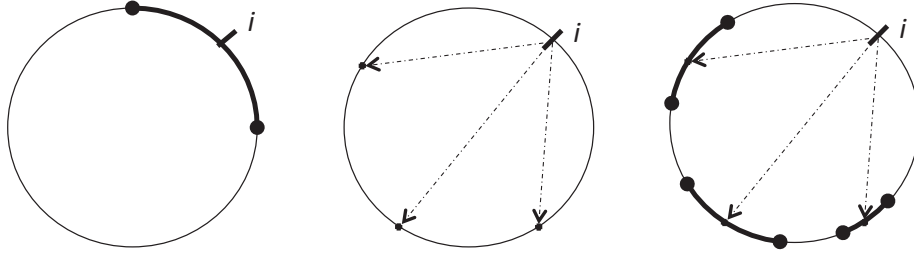


Figure 4: Illustration of participation choices under different assumptions on the participation costs $F_i(\{a_{i,1}; \dots; a_{i,N_i}\}, \Delta_i)$.

non-decreasing in N_i ; we set $b_1 = b_2 = 1$ without loss of generality.⁸

We make several remarks on specification (5). First we note that $F_i(\{i\}, 0) = 0$, so that participating only in the local market is costless. Second, the fact that f is increasing implies that investing in markets that are farther away (in the sense that the distance $d(a_{i,n}, i)$ from one's location is larger) is more costly than participating in markets that are close by. Third, increasing the total mass of markets in which the investor participates (Δ_i), while keeping the number (and midpoints) of intervals the same, incurs incremental rather than fixed costs. This captures the idea that expanding participation to contiguous markets is substantially less costly than participating in a market that is not adjacent to any of the markets where the investor has already decided to participate. Fourth, the fact that participation costs depend on the location of the investor implies that investors in different locations on the circle face different costs of participating in a given market j , depending on their proximity to that market. Finally, for technical convenience and ease of exposition, the costs of expanding the total measure of the interval are specified as independent of the structure of the midpoints and the lengths of the subintervals. This assumption is not crucial and can be easily relaxed.⁹

Even though one can specify more elaborate participation-cost structures, the one given by (5) is sufficiently flexible for our purposes. To better understand this cost structure it is useful to consider some special cases. In the first special case, the investor can only participate in an arc centered at her home location (depicted in the left plot of Figure 4).

⁸Since $f(0) = 0$, the first mid-point that an investor chooses is always her home location. Hence b_1 can be chosen arbitrarily, as $b_1 f(0) = 0$. Similarly, $b_2 = 1$ is without loss of generality, since we can always re-define $f^* = b_2 f$ and $b_N^* = \frac{b_N}{b_2}$ without changing the total cost F_i .

⁹Indeed, if one were to replace $g(\Delta_i)$ with $\sum_{n=1}^{N_i} g(\Delta_{i,n})$, that would introduce additional reasons for the cost structure to be non-convex, strengthening some of the conclusions of the paper.

This outcome follows from (5) if one sets $b_N f(x) = \infty$ and $g(y) < \infty$ for any $x > 0$, $y > 0$, and $N > 1$, so that the participation cost reduces to $F(\Delta_i) = \kappa g(\Delta_i)$. We note that in this special case the participation-cost structure is convex (unlike when the investor participates in disjoint intervals, which carries fixed costs). This convexity is quite attractive for illustrating some of the results of the paper, and therefore this special case will play an important role in our analysis. A second special case corresponds to situations in which an investor only participates on discrete points (middle plot of Figure 4). This outcome obtains from (5) if $b_N f(x) < \infty$ and $g(y) = \infty$ for any $y > 0$. In the general case $b_N f(x) < \infty$ and $g(y) < \infty$, the participation decision involves choosing the number of points N_i , the location of the midpoints $a_{i,n}$, and the length of the intervals $\Delta_{i,n}$ at each location. This situation is depicted in the right plot of Figure 4.

Besides the markets for risky shares, there is a market for zero-net-supply riskless bonds that pay one unit of wealth at time 1. Participation in the bond market is costless for everyone.

The participation costs act as deadweight costs that are paid out (i.e., reduce consumption) at time 1.

We remark in passing that our results would not be affected if, instead of making the participation decisions themselves, agents invested through frictionless and competitive intermediation. That is, we could introduce a competitive sector of intermediaries who incur the participation costs, choose optimal portfolios for their clients, and then charge them competitive fees to cover the participation costs.¹⁰

3.3 Individual maximization and equilibrium

To formalize an investor's decision problem we let P_i denote the price of a share in a market i , $P_B \equiv \frac{1}{1+r}$ the price of a bond, $dX_j^{(i)}$ the mass of shares in market j bought by an investor

¹⁰In a previous version of the paper, we consider such a model. Specifically, in this model a) investors don't have direct access to markets, but rather have to hire (competitive) intermediaries in their location to gain access to distant markets, b) the intermediaries attract clients by offering portfolios that maximize their welfare, and c) intermediaries charge clients fees to cover the participation costs that they incur. Not surprisingly, the lack of frictions between investors and intermediaries implies that the allocations and the prices in the two models are equivalent.

located in i ,¹¹ and $X_B^{(i)}$ the respective number of bonds. Then, letting

$$W_{0,i} \equiv \int_0^1 P_j dX_j^{(i)} + P_B X_B^{(i)}$$

denote the total financial wealth of an investor in location i , the budget constraint of an investor can be expressed as $W_{0,i} = P_i$.

We are now in a position to formulate the investor's maximization problem as

$$\max_{w_i^f, G^{(i)}, N_i, \vec{a}_i, \vec{\Delta}_i} E[U(W_{1,i})], \quad (6)$$

subject to the budget constraint, and

$$W_{1,i} = W_{0,i} \left(w_i^f (1+r) + (1-w_i^f) \int_0^1 R_j dG_j^{(i)} \right) - F_i, \quad (7)$$

where w_i^f is the fraction of $W_{0,i}$ invested in the risk-free security by an agent in location i , $G_j^{(i)}$ is a bounded-variation function with $\int_0^1 dG_j^{(i)} = 1$, which is constant in locations where the investor does not participate (i.e., $dG_j^{(i)} = 0$ in these locations), so that $dG_j^{(i)}$ captures the fraction of the risky component of the portfolio $(1-w_i^f)W_{0,i}$ invested in the share of stock j by a consumer located in i . Finally, $R_i \equiv \frac{D_i}{P_i}$ is the realized gross return on security i at time 1. We do not restrict G to be continuous, that is, we allow investors to invest mass points of wealth in some locations.

The definition of equilibrium is standard. An equilibrium is a set of prices P_i , a real interest rate r , and participation and portfolio decisions w_i^f , $G^{(i)}$, N_i , and $\{a_{i,1}, \dots, a_{i,N_i}, \Delta_{i,1}, \dots, \Delta_{i,N_i}\}$ for all $i \in [0, 1]$ such that: 1) w_i^f , $G^{(i)}$, N_i , and $\{a_{i,1}, \dots, a_{i,N_i}, \Delta_{i,1}, \dots, \Delta_{i,N_i}\}$ solve the optimization problem of equation (6), 2) financial markets for all stocks clear: $P_j = \int_{i \in [0,1]} (1-w_i^f) W_{0,i} dG_j^{(i)}$, and 3) the bond market clears, i.e., $\int_{i \in [0,1]} W_{0,i} w_i^f di = 0$.

By Walras' law, we need to normalize the price in one market. Since in the baseline model we abstract from consumption at time zero for parsimony, we normalize the price of the bond to be unity (i.e., we choose $r = 0$). We discuss consumption at more dates than

¹¹The function $X_j^{(i)}$ has finite variation. We adopt the natural convention that $X_j^{(i)}$ is continuous from the right and has left limits.

time one and an endogenously determined interest rate in Appendix A.1.

4 Solution and Its Properties

We solve the model and illustrate its properties in a sequence of steps. First, we discuss a frictionless benchmark, where participation costs are absent. Second, in order to build intuition we discuss the special case where the investor can only participate in an arc centered at her location, so that her participation decision amounts to choosing the length of that arc (the special case depicted on the left panel of Figure 4). We consider the general case, which allows for both contiguous and non-contiguous participation choices, in Appendix A.2.

4.1 A frictionless benchmark

As a benchmark, we consider first the case without participation costs: $F_i = 0$. In this case, the solution to the model is trivial. Every investor i participates in every market j . The first order condition for portfolio choice is

$$E [U'(W_{1,i}) (R_j - (1 + r))] = 0. \quad (8)$$

With the above first-order condition in hand, one can verify the validity of the following (symmetric) equilibrium: $w_i^f = 0$, and $G_j^{(i)} = G_j = j$ for all $(i, j) \in [0, 1) \times [0, 1)$ — i.e., investors in all locations i choose an equally weighted portfolio of every share $j \in [0, 1]$. Accordingly, $W_{1,i} = \int_0^1 D_j dj = 1$. Since in this equilibrium $W_{1,i} = 1$, the Euler equation (8) implies that

$$E(R_j) = 1 + r. \quad (9)$$

Combining (9) with the definition $R_j = \frac{D_j}{P_j}$ implies

$$P_j = \frac{E(D_j)}{1 + r} = \frac{1}{1 + r} = 1, \quad (10)$$

where the last equation follows from the normalization $r = 0$. The equilibrium in the frictionless case is intuitive. Since there are no participation costs, investors hold an equally weighted portfolio across all locations. Since — by assumption — there is no risk in the aggregate, the risk of any individual security is not priced.

4.2 Symmetric equilibria with participation costs

To facilitate the presentation of some of the key results, we focus on the special case depicted in the left plot of Figure 4, i.e., the case where the investor’s participation choice amounts to choosing the length Δ of an arc centered at her location. The associated participation cost is $\kappa g(\Delta)$.

To make the investor’s problem well-defined and interesting, we assume that the costs of participation increase sufficiently fast (and thus become infinite) as Δ approaches one:

Assumption 1 $\lim_{\Delta \rightarrow 1} g'(\Delta)(1 - \Delta)^4 = \infty$.

Assumption 1 helps ensure that an optimal Δ less than one exists for every investor.¹²

Before presenting results, we introduce a convention to simplify notation.

Convention 1 *For any real number x , let $\lfloor x \rfloor$ denote the floor of x , i.e., the largest integer weakly smaller than x . We henceforth use the term “location x ” (on the circle) to refer to the unique point in $[0, 1)$ given by $x \bmod 1 \equiv x - \lfloor x \rfloor$.*

With this convention we can map any real number to a unique location on the circle with circumference one. For example, this convention implies that the real numbers -0.8, 0.2, and 1.2 correspond to the same location on the circle with circumference one, namely 0.2. An implication of this convention is that any function h defined on the circle extends to a function \hat{h} on the real line that is periodic with period one, i.e., $\hat{h}(x) = \hat{h}(x + 1) = h(x \bmod 1)$.

¹²If $P < 1$, so that there is a risk premium, the limiting case $\Delta = 1$ is not well defined, since both the costs and the benefits of participation diverge to infinity as $\Delta \rightarrow 1$. Assumption 1 along with an application of L’Hôpital’s rule ensures that the cost component dominates the benefit component as $\Delta \rightarrow 1$, resulting in an optimal (interior) choice of $\Delta \in [0, 1)$. We also note that, if we introduce aggregate risk, then Assumption 1 can be replaced with the weaker $\lim_{\Delta \rightarrow 1} g(\Delta) = \infty$. Finally, no assumption is necessary in the presence of a leverage constraint, as in Section 5 (except that $g(\Delta) > 0$ for some $\Delta \in (0, 1)$).

mod 1). From now on, we adopt the convention that when we refer to a function h on the circle, we also refer to its extension to the real line.

We next introduce two definitions.

Definition 1 *The standardized portfolio associated with $G_j^{(i)}$ is the function L_j defined by $L_j = G_{i+j}^{(i)}$.*

The notion of a standardized portfolio allows us to compare portfolios of investors at different locations on the circle. For example, if all investors choose portfolios with weights that only depend on the distance d between their domicile and the location of investment, and these distance-dependent weights are the same for all investors, then these investors hold the same standardized portfolio.

Definition 2 *A symmetric equilibrium is an equilibrium in which all agents choose the same participation interval Δ , the same leverage w^f , and the same standardized portfolio.*

Due to the symmetry of the problem, it is natural to start by attempting to construct a symmetric equilibrium.

Proposition 1 *For any $\Delta \in (0, 1)$ define*

$$L_j^* \equiv \begin{cases} 0 & \text{if } j \in [-\frac{1}{2}, -\frac{\Delta}{2}) \\ j + \frac{1}{2} & \text{if } j \in [-\frac{\Delta}{2}, \frac{\Delta}{2}) \\ 1 & \text{if } j \in [\frac{\Delta}{2}, \frac{1}{2}) \end{cases} \quad (11)$$

and

$$\omega(\Delta) \equiv \text{Var} \left(\int_0^1 D_j dL_j^* \right) = \frac{\sigma^2}{12} (1 - \Delta)^3. \quad (12)$$

Finally, let Δ^* denote the (unique) solution to the equation

$$\kappa g'(\Delta^*) = -\frac{\gamma}{2} \omega'(\Delta^*) \quad (13)$$

and also consider the set of prices

$$P_i = P \equiv 1 - \gamma\omega(\Delta^*). \quad (14)$$

Then, assuming that a symmetric equilibrium exists, the choices $\Delta^{(i)} = \Delta^*$, $w_i^f = 0$, and $dG_j^{(i)} = dL_{j-i}^*$ constitute an equilibrium supported by the prices $P_i = P$.

Proposition 1 gives simple, explicit expressions for both the optimal portfolios and participation intervals. To understand how these quantities are derived, we take an individual agent's wealth $W_{1,i}$ from (7) and we assume that prices for risky assets are the same in all locations. Then, using $W_{0,i} = P_i = P$, $r = 0$, $R_j = \frac{D_j}{P}$, and $F_i = \kappa g(\Delta)$, we express $W_{1,i}$ as

$$\begin{aligned} W_{1,i} &= P \left(w_i^f + (1 - w_i^f) \int_0^1 R_j dG_j^{(i)} \right) - \kappa g(\Delta) \\ &= \left(Pw_i^f + (1 - w_i^f) \int_0^1 D_j dG_j^{(i)} \right) - \kappa g(\Delta). \end{aligned}$$

Because of exponential utilities and normally distributed returns, maximizing $EU(W_{1,i})$ is equivalent to solving

$$\max_{dG_j^{(i)}, \Delta, w_i^f} Pw_i^f + (1 - w_i^f) \int_0^1 ED_j dG_j^{(i)} - \frac{\gamma}{2} (1 - w_i^f)^2 \text{Var} \left(\int_0^1 D_j dG_j^{(i)} \right) - \kappa g(\Delta). \quad (15)$$

Noting that $ED_j = 1$, inspection of equation (15) shows that (for any w^f and Δ) the optimal portfolio is the one that minimizes the variance of dividends in the participation interval Δ . Since the covariance matrix of dividends is location invariant, the standardized variance-minimizing portfolio is the same at all locations. Solving for this variance-minimizing portfolio L_j is an infinite-dimensional optimization problem. However, because of the symmetry of the setup we are able to solve it explicitly, and equation (11) provides the solution. The optimal portfolio, L_j^* , corresponds to the distribution that minimizes the sum of the vertical distances to the uniform distribution on $[-\frac{1}{2}, \frac{1}{2})$, subject to the constraints that $L_j = 0$ for $j \in [-\frac{1}{2}, -\frac{\Delta}{2})$ and $L_j = 1$ if $j \in [\frac{\Delta}{2}, \frac{1}{2})$. The resulting optimized variance is given by $\omega(\Delta)$, where $\omega(\Delta)$ is defined in (12).

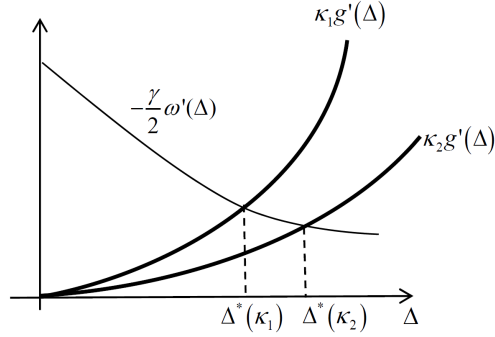


Figure 5: Illustration of the determination of Δ^* when a symmetric equilibrium exists. The figure illustrates that Δ^* increases when $\kappa_2 < \kappa_1$.

Accordingly, the agent's problem can be written more compactly as

$$V = \max_{\Delta, w_i^f} P w_i^f + (1 - w_i^f) - \frac{\gamma}{2} (1 - w_i^f)^2 \omega(\Delta) - \kappa g(\Delta). \quad (16)$$

The first-order conditions with respect to w_i^f , respectively Δ , are

$$1 - P = \gamma (1 - w_i^f) \omega(\Delta) \quad (17)$$

$$\kappa g'(\Delta) = -\frac{\gamma}{2} (1 - w_i^f)^2 \omega'(\Delta). \quad (18)$$

Since in a symmetric equilibrium market clearing requires $w_i^f = 0$, equation (17) becomes identical to (14) and (18) becomes equivalent to (13).

Equation (13) makes explicit the resolution of the tradeoff between participation costs and risk taking. Specifically, the participation interval is determined as the point where the marginal cost of participation, $\kappa g'(\Delta)$, is equal to the marginal benefit of participation, $-\frac{\gamma}{2} \omega'(\Delta)$.

Figure 5 illustrates this tradeoff by plotting the marginal cost from increasing Δ , namely $\kappa g'(\Delta)$, against the respective marginal benefit $-\frac{\gamma}{2} \omega'(\Delta)$. Since $g(\Delta)$ is convex, $g'(\Delta)$ is upward sloping. By contrast, the marginal benefit is declining, since $-\omega''(\Delta) = -\frac{\sigma^2}{2} (1 - \Delta) < 0$. Since $g'(0) = 0$, $\lim_{\Delta \rightarrow 1} g'(\Delta) = \infty$, $-\omega'(0) > 0$, and $\omega'(1) = 0$, the two curves intersect at some point $\Delta^* \in [0, 1)$.

Proposition 1 helps capture the economic mechanisms that underlie our model. Consider,

for instance, its implications for a reduction in the cost of accessing markets (i.e., a reduction in κ). As Figure 5 illustrates, such a reduction increases the degree of participation Δ and promotes portfolio flows across different locations. In turn, this increased participation improves risk sharing across different locations, which leads to higher prices of risky securities, $P = 1 - \gamma\omega(\Delta)$, and accordingly lower risk premia. By contrast, an increase in the costs of accessing risky markets leads to a lower Δ and a higher degree of concentration of risk. The resulting decline in the extent of risk sharing leads to a drop in the prices of risky assets and an increase in risk premia.

These mechanisms of the model capture the stylized facts summarized in Section 2. We highlight one aspect of our analysis: the extent of market integration and cross-location portfolio flows, on one hand, and the magnitude of risk premia, on the other, are intimately linked. By contrast, representative-agent approaches to the determination of risk premia are — by their construction — limited in their ability to explain the empirically prevalent joint movements in risk premia and portfolio movements, since the representative agent always holds the market portfolio and prices adjust so as to keep the agent content with her holdings.

We conclude this section by noting that in Appendix A.1 we extend the results obtained so far to an intertemporal version of the model with recurrent shocks to participation costs. That extension strengthens the comparative statics conclusions established above by showing that, in a dynamic setting with participation-cost shocks, the model produces a) a negative correlation between capital flows and excess returns, b) time variation in excess returns that is unrelated to expected dividend growth, aggregate output etc., and c) return correlation across locations that exceeds the respective correlation of dividends. Furthermore, the additional risk caused by variation in the participation costs is priced, resulting in an increased expected excess return.

Next we turn our attention to a central issue of our analysis, pertaining to the existence of a symmetric equilibrium.

4.3 Asymmetric, location-invariant equilibria: Leverage and the diversity of financial strategies

Proposition 1 contains the premise that a symmetric equilibrium exists. Surprisingly, despite the symmetry of the model setup, a symmetric equilibrium may fail to exist. Instead, the market equilibrium may involve different choices (leverage ratios, portfolios of risky assets, wealth allocations, etc.) for agents in the same location, even though these agents have the same preferences and endowments and are allowed to make the same participation and portfolio choices.

These claims are explained by the observation that the necessary first-order conditions resulting in the prices $P_i = P = 1 - \gamma\omega(\Delta^*)$ are not generally sufficient. We now take a closer look at whether a symmetric equilibrium exists. Specifically, we fix the price $P = 1 - \gamma\omega(\Delta^*)$ of Proposition 1 and investigate whether (and under what conditions) the choices $w^f = 0$ and $\Delta = \Delta^*$ are indeed optimal for the investor.

To answer this question, we consider again the maximization problem (16). Taking $P = 1 - \gamma\omega(\Delta^*)$ as given, substituting into (17), and re-arranging implies that if investors allocate their wealth over a participation interval Δ (potentially different from Δ^*), then

$$1 - w^f = \frac{\omega(\Delta^*)}{\omega(\Delta)}. \quad (19)$$

Equation (19) contains an intuitive prediction. An investor allocating her wealth over a span $\Delta > \Delta^*$ is facing the same average returns, but a lower variance $\omega(\Delta)$, and hence a higher Sharpe ratio compared to an investor allocating her wealth over an interval of size Δ^* . Accordingly, the former investor finds it optimal to leverage her portfolio. This is reflected in equation (19), which states that $w^f < 0$ ($w^f > 0$) whenever $\Delta^* < \Delta$ ($\Delta^* > \Delta$).

An interesting implication of (18) is that the marginal benefit of an increased participation interval becomes larger with leverage. This is intuitive since increased leverage implies a more volatile wealth next period and hence a higher marginal benefit of reducing that variance by increasing Δ . In short, the choices of Δ and of leverage $1 - w^f$ are *complements*.

This complementarity can lead to an upward sloping marginal benefit of increased par-

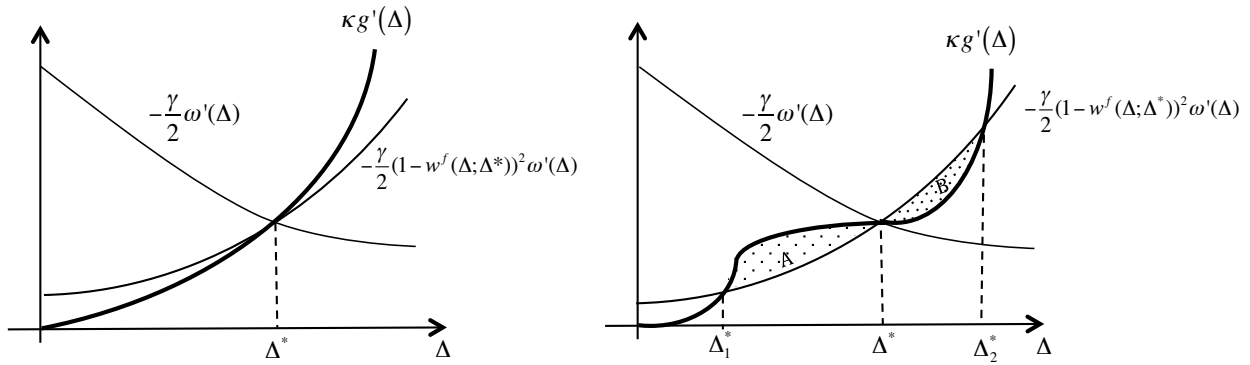


Figure 6: Illustration of the marginal benefit curve $-\frac{\gamma}{2}\left(\frac{\omega(\Delta^*)}{\omega(\Delta)}\right)^2\omega'(\Delta)$, the marginal cost curve $\kappa g'(\Delta)$ and the marginal benefit curve $-\frac{\gamma}{2}\omega'(\Delta)$ restricting $w^f = 0$. The left plot illustrates a case where a symmetric equilibrium exists whereas the right plot illustrates a case where a symmetric equilibrium fails to exist.

ticipation. Indeed, with our cash-flow specification, substituting (19) into (18) gives

$$\kappa g'(\Delta) = -\frac{\gamma}{2}\left(\frac{\omega(\Delta^*)}{\omega(\Delta)}\right)^2\omega'(\Delta). \quad (20)$$

Using the fact $\omega(\Delta) = \frac{\sigma^2}{12}(1-\Delta)^3$, the right hand side can be expressed as $-\frac{\gamma}{2}\left(\frac{\omega(\Delta^*)}{\omega(\Delta)}\right)^2\omega'(\Delta) = \frac{\gamma\sigma^2}{8}(1-\Delta^*)^6(1-\Delta)^{-4}$, which is increasing in Δ .

Figure 6 helps illustrate these notions. The figure depicts the marginal cost curve $\kappa g'(\Delta)$, the marginal benefit curve $-\frac{\gamma}{2}(1-w^f(\Delta;\Delta^*))^2\omega'(\Delta) = -\frac{\gamma}{2}\left(\frac{\omega(\Delta^*)}{\omega(\Delta)}\right)^2\omega'(\Delta)$, and also the curve $-\frac{\gamma}{2}\omega'(\Delta)$, i.e., the marginal benefit of participation fixing $w^f = 0$. The point where all three curves intersect corresponds to the point $\Delta = \Delta^*$. The left plot of Figure 6 illustrates a case where a symmetric equilibrium exists, whereas the right plot illustrates a case where a symmetric equilibrium fails to exist. The difference between the two plots is the shape of $g'(\Delta)$. In the left plot $g'(\Delta)$ intersects the marginal benefit curve only once, namely at Δ^* . For values smaller than Δ^* , the marginal benefit of participation is above the marginal cost and vice versa for values larger than Δ^* . Hence, in this case Δ^* is indeed the optimal choice for the investor.

This is no longer the case in the right plot. Here the marginal benefit curve intersects the marginal cost curve three times (at Δ_1^* , Δ^* , and Δ_2^*). Since the marginal benefit is below the

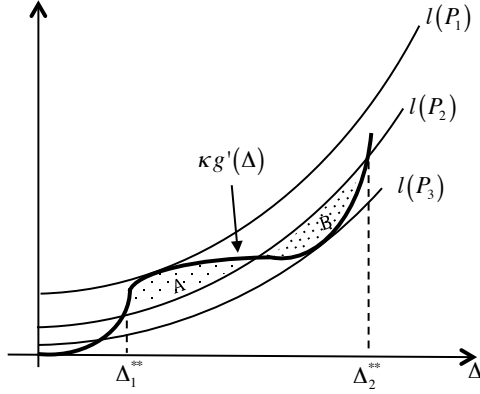


Figure 7: Illustration of an asymmetric equilibrium involving mixed strategies. The price adjusts so that area A is equal to area B . Accordingly, an investor is indifferent between Δ_1^{**} and Δ_2^{**} . Here, $P_3 < P_2 < P_1 < 1$.

marginal cost for values of Δ that are smaller and “close” to Δ^* , while the marginal benefit is larger than the marginal cost for values of Δ that are larger and “close” to Δ^* , Δ^* is a *local minimum*, and hence a suboptimal choice. By contrast the points Δ_1^* and Δ_2^* are local maxima. (In this particular example the point Δ_1^* is the global maximum, since the area A is larger than the area B .) The fact that Δ^* is not a maximum implies that there does not exist a symmetric equilibrium, since in a symmetric equilibrium it would have to be the case that $w^f = 0$.

The fact that there does not exist a symmetric market equilibrium implies that one should look for equilibria where investors in the same location make different choices, even though they have the same preferences, endowments, and information. Figure 7 presents a simple graphical illustration of such an equilibrium in the context of the example depicted on the right plot of Figure 6.

Specifically we illustrate the construction of an equilibrium that features the same price $P_i = P$ for all markets, but where a fraction π of investors in every location i choose (w_1^f, Δ_1) , while the remaining fraction $(1 - \pi)$ of agents choose (w_2^f, Δ_2) . We introduce a function l that captures the marginal benefit of participation as a function of Δ and P :

$$l(\Delta, P) \equiv -\frac{\gamma}{2} (1 - w^f(\Delta, P))^2 \omega'(\Delta) = -\frac{\gamma}{2} \left(\frac{1 - P}{\gamma} \right)^2 \frac{\omega'(\Delta)}{\omega^2(\Delta)}, \quad (21)$$

where the last equation follows from (17).

Figure 7 depicts the function $l(\Delta, P)$ for three values $P_1 < P_2 < P_3 < 1$. A first observation is that as P declines from P_3 to P_1 , the curve $l(\Delta, P)$ shifts up. Moreover there exists a level P_2 and associated values Δ_1^{**} and Δ_2^{**} for which the area “A” equals the area “B”, so that investors are indifferent between choosing Δ_1^{**} and Δ_2^{**} . Fixing these values of P, Δ_1^{**} , and Δ_2^{**} , we can determine the values of w_1^f and w_2^f from (17). As part of the proof of Proposition 2 (in the appendix), we show that these values of w_1^f and w_2^f satisfy $1 - w_1^f < 1 < 1 - w_2^f$. In order to clear the bond market, π has to be so that $\pi w_1^f + (1 - \pi)w_2^f = 0$, i.e., $\pi = \frac{w_2^f}{w_2^f - w_1^f} \in (0, 1)$.¹³ We also show in the appendix that for this value of π all markets for risky assets clear as well.

The next proposition generalizes the insights of the above illustrative example. It shows the existence of an asymmetric equilibrium, when a symmetric equilibrium fails to exist.

Proposition 2 *When the cost function g is such that a symmetric equilibrium fails to exist, there exists an asymmetric equilibrium. Specifically, there exist (at least) two tuples $\{\Delta_k, w_k^f\}$, $k \geq 2$, and $\pi_k > 0$ with $\sum_k \pi_k = 1$ and $\sum_k \pi_k(1 - w_k^f) = 1$, such that in every location i a fraction π_k of agents choose the interval, leverage, and portfolio combination $\{\Delta_k, w_k^f, dL^{(k)}\}$, where $dL^{(k)}$ is the measure given in (11) for $\Delta = \Delta_k$.*

Hence, the model predicts that the complementarity between leverage and the incentive to increase participation in risky markets may result in a non-concave objective function for the determination of Δ . In such situations a symmetric equilibrium can fail to exist. Instead, the market equilibrium features a diverse set of financial strategies, with some investors pursuing high-cost, high-Sharpe-ratio, high-leverage strategies and some investors pursuing low-cost, low-Sharpe-ratio, no-leverage strategies. The first type of strategies is akin to the sort of strategies offered by hedge funds, while the second type of strategies is akin to those offered by mutual funds. It is useful to underscore that this diversity of financial strategies obtains despite the facts that investors are identical and participation costs are convex.

When the costs of participation are non-convex (for instance, because investors may choose non-contiguous participation locations, as in Appendix A.2), there is an additional

¹³ $\pi \in (0, 1)$ since $w_2^f < 0$ and $w_1^f > 0$.

and conceptually distinct reason why the investor’s objective is non-concave, leading to the adoption of diverse financial strategies.¹⁴

In summary, this section has established that the non-concavities in investors’ objective functions introduce diversity in the optimal participation-leverage combinations chosen by different investors. Importantly, the equilibrium price plays a key role in determining the participation-leverage combinations that keep investors indifferent between different strategies. In the next section we identify a tension that arises when we introduce collateral constraints, so that the price simultaneously affects investors’ indifference relations and their feasible leverage-participation combinations.

5 Collateral Constraints and Price Crashes

The previous section shows that equilibrium outcomes may feature heterogeneous investment strategies in every location despite the fact that investors are identical. An important aspect of these heterogeneous strategies is that they require different degrees of leverage. Even though in the model leverage can be chosen freely, in reality leverage is limited by collateral constraints. In this section we discuss the implications of such constraints. We present a simple example to show that even small increments in participation costs may lead to an abrupt drop in prices, along with a sudden reversal in cross-location capital flows and de-leveraging.

To model collateral constraints, we let $\chi \in [0, 1)$ denote a “haircut” parameter and require

$$\left[(1 - \chi) \int_0^1 P_j dX_j^{(i)} - F_i \right] + X_B^{(i)} \geq 0. \quad (22)$$

Equation (22) stipulates that the borrowing capacity of an investor (the first term — inside the square brackets — of (22)) do not exceed the amount the investor actually borrows (the second term of (22)). The investor’s borrowing capacity is computed as the market value of

¹⁴A classic example, which also features different choices by ex-ante identical investors, due to fixed (hence non-convex) information costs, is Grossman and Stiglitz (1980). A difference between Grossman and Stiglitz (1980) and our setup, which becomes pertinent in the next section, is that the price adjusts to keep investors indifferent between different combinations of leverage and participation, while in Grossman and Stiglitz (1980) investors decide on paying information acquisition costs before observing the price.

the collateral net of the haircut and of the obligation the investor has already assumed by committing to pay the participation costs in period 1.

The collateral constraint (22) is pervasive in reality, extensively used in the literature, and analytically convenient for our purposes. Therefore, we focus on the implications of this constraint in the body of the paper. In Appendix D we analyze an alternative, endogenous version of the constraint (22) motivated by the desire to prevent default in all states of nature. We show that the key insights of the present section remain the same.

To further streamline the exposition, in this section we use a stylized specification of participation costs that allows analytical computations. Specifically, we postulate the cost function depicted in the middle graph of Figure 4. In that case, the optimal participation choice amounts to choosing the location of the points a_i and the number of the discrete points N .¹⁵ To keep computations as simple as possible, we assume furthermore that $b_N = \infty$ for $N > 2$, so that only choices involving $N \leq 2$ are feasible.¹⁶ Accordingly, an investor's problem comes down to choosing $N = 1$ (and paying no participation costs) or $N = 2$ and the distance d from her home location, resulting in participation costs equal to $\kappa f(d)$. (See Appendix A.2 for an extension to multiple locations.) In an effort to further streamline the analysis, we assume that $f'(d) = 0$. In that case the investor faces only a fixed cost of $\kappa f(0)$ when choosing $N = 2$, but can choose d freely (i.e., without incurring additional cost).

We start by analyzing equilibrium prices in this economy when we don't impose the constraint (22) on investors' decisions.

Proposition 3 *Assume that the borrowing constraint is not imposed on investors' decisions, and also assume that $\kappa \in (\kappa_1, \kappa_2)$, where $\kappa_1 \equiv \frac{1}{128} \frac{\gamma \sigma^2}{f_0}$, $\kappa_2 \equiv \frac{1}{8} \frac{\gamma \sigma^2}{f_0}$. Then there exists an asymmetric equilibrium in which a fraction $\pi > 0$ of the investors choose $N = 1$, while the*

¹⁵Recall that in this case there is a minimum "fixed" cost that one needs to pay for every new location that she chooses given by $\kappa b_N \lim_{x \rightarrow 0} f(x) > 0$.

¹⁶The condition $b_N = \infty$ can be substantially weakened. Indeed all that is needed for $N = 1$ or $N = 2$ to be the only feasible choices is that $\kappa b_N \lim_{x \rightarrow 0} f(x) > 1$ for $N > 2$. To see this, use the normalization $P_B = 1$ along with the budget constraint to obtain $P_i = W_{0,i} = \int_0^1 P_j dX_j^{(i)} + X_B^{(i)} = (1 - \chi) \int_0^1 P_j dX_j^{(i)} - F_i + X_B^{(i)} + F_i + \chi \int_0^1 P_j dX_j^{(i)} \geq F_i + \chi \int_0^1 P_j dX_j^{(i)}$, where the inequality follows from (22). Noting that $P_j \leq 1$ and f is non-decreasing, the only feasible choices are $N = 1$ or $N = 2$ if $\kappa b_N \lim_{x \rightarrow 0} f(x) > 1$ for $N > 2$.

rest choose $N = 2$ and $d = \frac{1}{2}$. The equilibrium price in all locations is

$$P(\kappa) = 1 - \sigma \sqrt{\frac{\gamma \kappa f_0}{18}}, \quad (23)$$

and the choices w_i^f satisfy $w_1^f > 0$ and $w_2^f < 0$. Finally,

$$\frac{dP}{d\kappa} = -\frac{f_0}{w_1^f - w_2^f} < 0. \quad (24)$$

For values of κ smaller than κ_1 or larger than κ_2 equilibria are symmetric with everyone choosing $N = 2$ and $d = \frac{1}{2}$ or everyone choosing $N = 1$, respectively. Hence, without imposing the constraint (22), the left plot of Figure 8 gives a visual depiction of $P(\kappa)$, which is a continuous function of κ . Even though investors make essentially discrete choices, the fraction of investors choosing each type of strategy is continuous in κ , rendering the equilibrium price continuous.

When imposing the constraint (22), however, a different situation arises, depicted in the right plot of Figure 8. The downward-sloping dotted line graphs the price $P(\kappa)$ that would obtain in the absence of the constraint (22), i.e., it is identical to the solid line of the left plot. The upward-sloping dotted line depicts the locus (κ, P) for which

$$\kappa f_0 = P \left(1 - \chi(1 - w_2^f) \right), \quad (25)$$

where $w_2^f(P)$ is given by (50). For these points the constraint (22) binds with equality.¹⁷

For values of κ and P that lie above and to the left of the upward-sloping dotted line, the constraint (22) is not binding; hence the equilibrium price (the solid line) coincides with the downward-sloping dotted line for values of κ smaller than the value marked as κ_c in Figure 8. However, for values of κ greater than κ_c , the downward-sloping dotted line no longer describes the equilibrium price.

The following Lemma characterizes the behavior of P as a function of κ .

¹⁷Combining the investor's budget constraint with (22) and repeating the same steps as in footnote 16 gives $W_{0,i} \geq F_i + \chi \int_0^1 P_j dX_j^{(i)}$. Evaluating this inequality with $P_i = P_j = P = W_{0,i}$, $P_B = 1$, $F_i = \kappa f_0$, and $\int_0^1 dX_j = 1 - w^f$ gives (25).

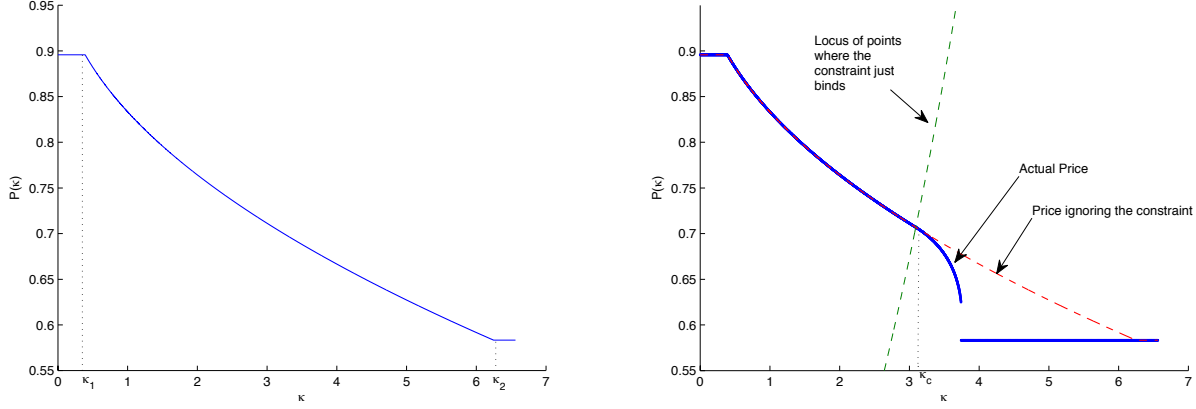


Figure 8: The left plot depicts the price $P(\kappa)$ assuming the leverage constraint is not binding. The downward-sloping dotted line on the right plot plots the same quantity, along with the actual price (solid line) and the locus of points such that the leverage constraint is just binding (upward sloping line). Hence for points above and to the left of the upward sloping line the constraint is not binding, whereas it is binding for points below and to the right. Parameters used in this example: $\gamma = 5$, $f_0 = 0.1$, $\chi = 0.2$, $\sigma = 1$.

Lemma 2 *Let $\lambda \geq 0$ denote the Lagrange multiplier associated with the constraint (22). Then assuming that*

$$w_1^f - w_2^f - \lambda \left[1 - \chi \left(1 - w_2^f \right) \right] > 0, \quad (26)$$

we have that

$$\frac{dP}{d\kappa} = - \frac{(1 + \lambda) f_0}{w_1^f - w_2^f - \lambda \left[1 - \chi \left(1 - w_2^f \right) \right]} < 0. \quad (27)$$

Comparing (24) with (27) shows that the slope $\frac{dP}{d\kappa}$ in the presence of a constraint is steeper than the respective expression in the absence of the constraint (22), since

$$- \frac{(1 + \lambda) f_0}{w_1^f - w_2^f - \lambda \left[1 - \chi \left(1 - w_2^f \right) \right]} < - \frac{f_0}{w_1^f - w_2^f}.$$

An important assumption for $\frac{dP}{d\kappa}$ to be well-defined (in the presence of the constraint) is contained in equation (26). This assumption is always satisfied in an asymmetric equilibrium as long as the Lagrange multiplier λ is sufficiently close to zero, since in any asymmetric

equilibrium we must have $w_1^f > 0$ and $w_2^f < 0$. Hence, condition (26) holds as long as κ is sufficiently close to κ_c , the minimal value of κ for which the leverage constraint binds.

As κ keeps increasing above κ_c , the constraint becomes progressively more binding, and possibly up to the point where (26) doesn't hold. In that case there may be no marginal change in P that can keep investors indifferent between the alternative financial strategies and $P(\kappa)$ can exhibit a discontinuity, accompanied by a change in the nature of equilibrium (from asymmetric to symmetric).

Figure 9 helps illustrate such a discontinuity in the context of the numerical example of Figure 8. For each subplot we fix a value of κ . Given that value of κ , the line denoted “Indifference” in each subplot depicts combinations of P and possible choices of $(1 - w_2^f)$ such that the indifference relation $V_1 = V_2$ holds.¹⁸ Similarly, the line denoted “Constraint” depicts combinations of P and $1 - w_2^f$ such that the constraint (25) holds as an equality. Accordingly, all the points that lie above that line are admissible combinations of P and $1 - w_2^f$. Finally, the line denoted “FOC” depicts the combination of points that satisfy the (unconstrained) first-order condition for leverage

$$1 - P = \gamma (1 - w_2^f) \omega^{(N=2)},$$

where $\omega^{(N=2)} = \frac{\sigma^2}{48}$ is the minimal variance attainable for an investor choosing to participate in $N = 2$ locations.

The intersection of the lines “Indifference” and “FOC” corresponds to the unconstrained equilibrium price (and the associated leverage w_2^f) of equation (23). The left plot depicts the case $\kappa = \kappa_c$. In that case the constraint just becomes binding, and all three lines intersect at the same point. For (moderately) higher values of κ , illustrated in the middle panel, the constraint binds actively. The intersection of “Indifference” and “FOC” no longer depicts an equilibrium. Instead, investors pursuing high leverage strategies reduce the magnitude of their risky positions, and equilibrium prices drop to the point where the constraint is satisfied (i.e., up to the point where the line “Indifference” intersects the line “Constraint”).

The right-most plot shows a case in which κ is large enough that there is no point of

¹⁸See equation (49) in the appendix for a formal statement of the indifference relation.

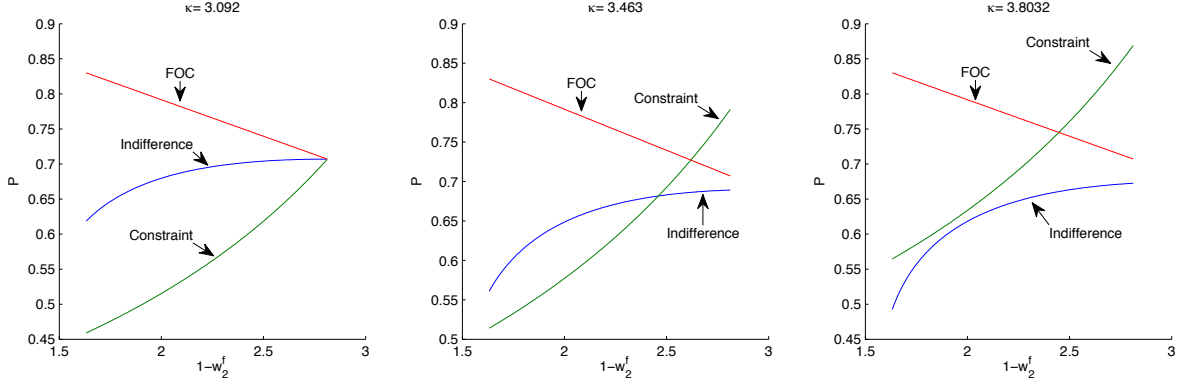


Figure 9: For three different values of the cost parameter κ , the three plots depict combinations of price (P) and leverage ($1 - w_2^f$) so that (i) investors are indifferent between adopting no- and high-leverage strategies (line labeled “Indifference”); (ii) investors’ choices of leverage ($1 - w_2^f$) are (unconstrained) optimal given P (line labeled “FOC”); and (iii) the leverage constraint just binds (line labeled “Constraint”). In each plot, points to the left and above the line labeled “Constraint” are admissible. The intersection of the lines “FOC” and “Indifference” corresponds to the combination of P and $1 - w_2^f$ that would prevail in an equilibrium without the leverage constraint. Similarly, the intersection of the lines “Indifference” and “Constraint” characterize an asymmetric equilibrium with a binding leverage constraint (if one exists). Parameters are identical to Figure 8.

intersection between the lines “Indifference” and “Constraint”. At that point there can be no asymmetric equilibrium. The equilibrium becomes symmetric, i.e., the price drops to the level that obtains when everyone chooses $N = 1$ and $w^f = 0$.

This example illustrates that — when the leverage constraint binds — the price in an asymmetric equilibrium may drop discontinuously to the level associated with a symmetric equilibrium, even in response to incremental changes in the cost parameter κ .

To understand intuitively why prices react so abruptly, it is helpful to start from the basic mechanism at play when κ increases and the constraint (22) isn’t binding: At existing prices, an increase in κ induces investors to walk away from high-leverage, increased-market-participation, high-cost strategies. The resulting loss in aggregate participation lowers prices, which restores the relative attractiveness of high-cost, high-leverage strategies. The end result is a marginally lower price level.

However, when the constraint binds, the decline in prices may not have the same beneficial effect on the attractiveness of levered strategies, since it also lowers the value of collateral that investors are endowed with. This necessitates reductions in leverage, which results in

even smaller overall participation in risky markets, further reductions in prices, etc. These mutually reinforcing effects can lead to a rapid drop, and eventually a discontinuous jump, in prices, whereby investors de-leverage abruptly and cross-location investment drops to zero.

6 Reduced Participation in a Subset of Locations, Entanglement, and Contagion

In the previous sections we assumed the same cost structure for all investors, regardless of location. This allowed us to focus on (potentially asymmetric but) location-invariant equilibria, which entail the same price in all locations. In this section we relax this assumption in order to study the effect of an adverse shock to financial participation in a subset of locations. The challenge in analyzing the model with location-specific costs is that an entire price function on $[0, 1)$ has to be computed, rather than a single value. To present the key insights of the model with location-specific costs, we restrict ourselves to a specific setup that is sufficiently simple to analyze, yet rich enough to illustrate how a reduction in market participation in a subset of risky markets can propagate across all markets.

Specifically, we consider the following stylized setup: for some positive $k < 1$ market access in locations $[-\frac{k}{2}; \frac{k}{2}]$ “breaks down”. Specifically, we assume that in these “affected” locations the cost parameter κ becomes infinite, so that investors choose to participate only in the bond market and the market for the local risky claim, but not in other markets. In the rest of the locations investors choose to participate in a single interval of length $\bar{\Delta}$, centered at their “home location” — this would be the outcome of a cost structure in these locations involving, e.g., $\kappa < \infty$, $b_N = \infty$ for $N > 1$, $g'(\bar{\Delta}) = 0$ for $\Delta < \bar{\Delta}$, and $g'(\Delta)$ sufficiently large for $\Delta > \bar{\Delta}$.

We solve this heterogeneous-participation-costs version of the model in the appendix. First, we calculate analytically the optimal demand of every investor given a price function (Lemma 4), and then aggregate the demands to solve for prices. We are unable to find a closed-form solution for the prices, but we can characterize the solution in terms of a linear integro-differential equation with delay, which can be solved numerically as easily as a matrix

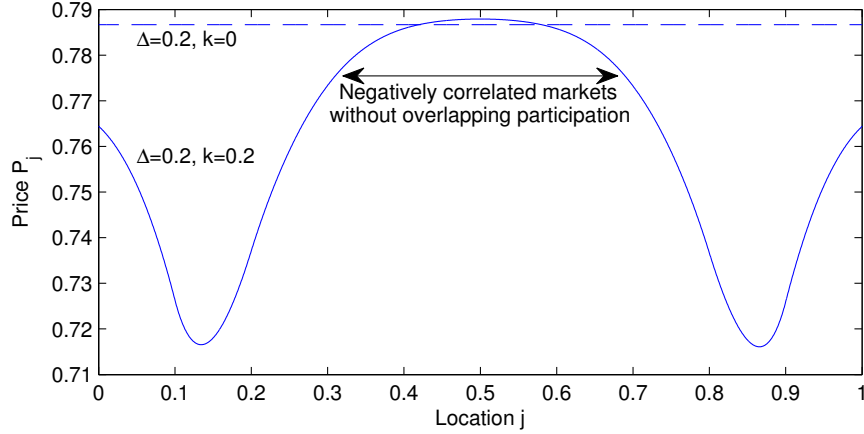


Figure 10: Contagion effects. The price function P_j depicts the price in different locations when $k = 0$ and $k = 0.2$. We set the length of the participation arc of “unaffected” investors to $\Delta = \bar{\Delta} = 0.2$. The locations between the arrows feature both negative correlation of dividends and no overlap between the participants in these markets and the set $A = [-\frac{k}{2}, \frac{k}{2}]$.

inversion problem.

To obtain a visual impression of the solution, Figure 10 depicts the price P_j for $j \in [0, 1]$ and compares it with the (symmetric) equilibrium price $P^*(\bar{\Delta})$ that would prevail if all agents in all locations chose a participation interval with length equal to $\Delta = \bar{\Delta}$. There are several noteworthy facts about Figure 10. As one might expect, prices in the set $A \equiv [-\frac{k}{2}, \frac{k}{2}]$ are lower than $P^*(\bar{\Delta})$. The more important fact is that prices in all other markets are affected, as well. This holds true even in markets in the interval $B \equiv [\frac{k}{2} + \bar{\Delta}, 1 - \frac{k}{2} - \bar{\Delta}]$, i.e., in locations that are at a larger distance than $\bar{\Delta}$ from the set A , so that agents in the set A would not be directly trading risky assets with agents in the set B , even if they were allowed to choose the same participation interval $\bar{\Delta}$ as everyone else. A third observation is that the prices in almost all markets are lower than $P^*(\bar{\Delta})$. Indeed, this reduction in price can happen even in markets with dividends that have a zero or negative correlation with the dividends of any risky security in A , as the figure illustrates.

The intuition behind Figure 10 is the following. By assumption, investors in the set A concentrate all their demand for risky assets in their home location. Hence, compared to the case in which everyone participates in an interval of length $\bar{\Delta}$ centered at their location, there is now lower demand for risky securities in locations neighboring the set A . The lower demand for risky securities in these locations leads to lower prices, which attracts

demand from locations adjacent to the neighborhood of A . By tilting their portfolio towards these locations, investors remove demand for risky securities in locations even farther from A . Accordingly, prices in these farther locations need to drop in order to attract investors from locations adjacent to the neighbourhood of the neighbourhood of A , who also tilt their portfolios and so on.

This chain reaction implies that all locations are affected in equilibrium. In fact, even some prices in locations close to point $\frac{1}{2}$, the farthest from A , drop, as the following result states.¹⁹

Proposition 4 *Assume $\bar{\Delta} + k < 1$ and that $\frac{1}{2} = \arg \max_{j \in [\frac{1}{2} - \bar{\Delta}, \frac{1}{2} + \bar{\Delta}]} P_j$. Then there exists a positive-measure subset $D \subset [\frac{1}{2} - \bar{\Delta}; \frac{1}{2} + \bar{\Delta}]$ such that $P(x) < P^*(\bar{\Delta})$ for all $x \in D$.*

Proposition 4 helps formalize the notion that the drop in prices caused by restricting market access for investors in the set A can permeate even a neighborhood of radius $\bar{\Delta}$ around the maximal-price location.

The extent of the change in risk premia in the various locations depends on the distance from the arc A , the length of the arc A , and most interestingly, on the length of the participation arc $\bar{\Delta}$. Figure 11 illustrates these statements. If $\bar{\Delta}$ is small, so that investors' portfolios are heavily exposed to risks in their vicinity, then even a small portfolio tilt towards a nearby location requires a relatively high risk compensation. By contrast, if investors' portfolios are invested across a broad range of locations, then they are more willing to absorb risks in their vicinity.

In summary, a surprising result of our analysis is that contagion occurs due to *limited*, rather than excessive, integration of risky markets. Indeed, contagion may become stronger the weaker the integration of risky markets. Finally, in model simulations not reported here due to space limitations, we have also considered the effect of local participation shocks when investors participate in multiple disjoint intervals, obtaining similar conclusions.²⁰

¹⁹It is easy to show that, in the general case when a subset of investors are limited to a smaller investment set than the original arc of length $\bar{\Delta}$, and therefore in the setting of this section, the average risky-asset price in the economy declines.

²⁰In such situations, the direct impact of a local shock to participation is felt not only in the neighboring locations of the affected locations, but also in the other subintervals where the affected investors used to participate. This sets off multiple "waves" of contagion across the circle (i.e., Figure 11 features more than two local minima).

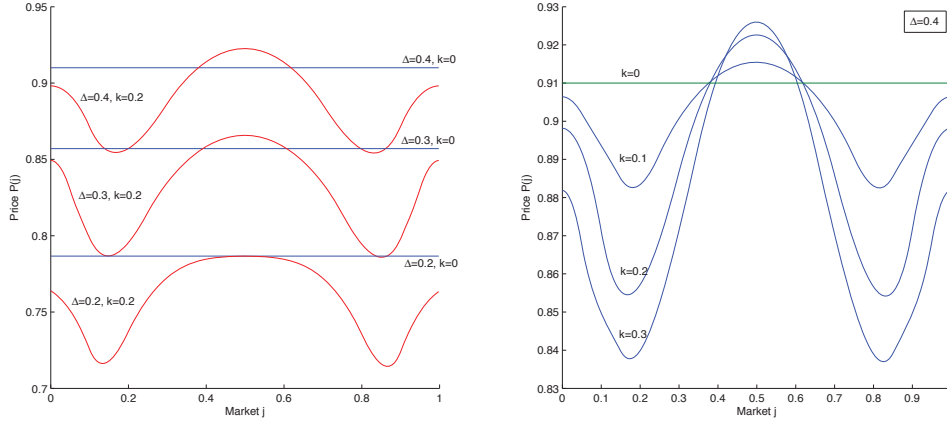


Figure 11: Contagion effects for various choices of $\bar{\Delta}$ and k . In the figures j denotes a specific market, k controls the length of the region with restricted access to markets, while Δ refers to $\bar{\Delta}$, that is the length of the participation arc for investors who can participate in “distant” markets.

7 Conclusion

We introduce a novel, location-invariant, and tractable structure of shocks on a circle that allows us to model the tradeoffs between integration and risk diversification on the one hand and participation costs on the other. The focus of the analysis is on the private incentives to incur costs to participate in various asset markets, and on the interdependence (“entanglement”) of markets in the presence of limited integration.

Our main findings can be grouped in three broad categories. First, participation decisions and leverage decisions are complements. Investors who choose to participate in a large number of markets also choose high degrees of leverage. The interaction between leverage decisions and participation routinely makes an investor’s optimization problem non-concave. Such non-concavities may imply a heterogeneous market structure. Despite the fact that all investors are identical, and even when participation costs are convex, some investors may opt for non-levered, low-participation investment strategies with others opting for levered, high-participation strategies.

Second, the non-concavity of an investor’s participation problem can lead to an abrupt change in the nature of equilibrium. Even small variations in participation costs can lead to rapid drops in leverage, prices and market integration when the ability to leverage is limited by the availability of collateral.

Third, asset markets exhibit increased interdependence despite — in fact, because of — limited market integration. Limitations to market access in one subset of locations propagates across markets. Surprisingly, the impact of these local shocks on other markets may be strengthened when markets are less integrated. In an intertemporal version of the model, the joint determination of market integration and equilibrium prices implies price and return correlations that exceed those of the underlying dividends.

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Appendix

A Extensions

A.1 An infinite horizon version of the baseline model

In this section we develop an intertemporal version of the model. The intertemporal version allows us to extend the intuitions of our comparative statics exercises to a framework where the shocks to the participation technology are recurrent.

Specifically, we keep the key assumptions of the baseline model (Section 4.2), and in particular the assumption that investors only participate in an arc surrounding their home location. However, we assume that investors maximize expected discounted utility from consumption

$$-\sum_{t=0}^{\infty} \beta^t E_0 [e^{-\gamma c_{t,i}}],$$

where t denotes (discrete) calendar time. We assume that dividends at location $i \in [0, 1]$ are given by

$$D_{t,i} = (1 - \rho) \sum_{k=-\infty}^t \rho^{t-k} \varepsilon_{k,i}, \quad (28)$$

where $\rho < 1$,

$$\varepsilon_{t,i} \equiv 1 + \sigma \left(B_i^{(t)} - \int_0^1 B_j^{(t)} dj \right), \quad (29)$$

and $B_i^{(t)}$ denotes a family of Brownian Bridges on $[0, 1]$ drawn independently across times $t \in \{-\infty, \dots, 1, 2, \dots, \infty\}$.

We make a few observations about this dividend structure. First, we note that equation (29) coincides with equation (2). Accordingly, $\int_0^1 \varepsilon_{t,i} di = 1$, and therefore, equation (28) implies that $\int_0^1 D_{t,i} = 1$ for all t , so that the aggregate dividend is always equal to one. Second, dividends at individual locations follow AR(1) processes, since equation (28) implies

$$D_{t,i} = (1 - \rho) \varepsilon_{t,i} + \rho D_{t-1,i}. \quad (30)$$

Moreover, since (29) coincides with (2), the increments of two dividend processes at two locations i and j have the covariance structure of equation (3).

In terms of participation decisions, we keep the same cost assumption as in the baseline model and further assume that investors participate in a single interval of length $\Delta_t^{(i)}$ centered at their “home” location. Participation costs are paid period by period in advance of trading. Specifically, an investor’s intertemporal budget constraint is given by

$$c_{t,i} + F_t \left(\Delta_t^{(i)} \right) + \int_0^1 P_{t,j} dX_{t,j}^{(i)} + P_{B,t} X_{B,t}^{(i)} = \int_0^1 (P_{t,j} + D_{t,j}) dX_{t-1,j}^{(i)} + X_{B,t-1}^{(i)}. \quad (31)$$

We note that in equation (31) we allow the entire cost function $F_t(\Delta) = \kappa_t g_t(\Delta)$ to be differ-

ent across different periods in order to capture the effect of repeated shocks to the participation technology.

This intertemporal version of the model presents a challenge that is absent in a static framework: If the interest rate varies over time, then the value function of an agent is not exponential in wealth. In fact, a closed-form expression for the value function most likely does not exist. Furthermore, once the value function is no longer exponential, portfolios are no longer independent of wealth, and hence the entire wealth distribution matters — an infinite-dimensional state variable.

In order to maintain the simple structure of the solution, therefore, we make necessary assumptions to achieve a constant interest rate in the presence of random participation costs, while safeguarding market clearing for bond markets, risky-asset markets, and consumption markets.²¹ The following proposition states that it is possible to achieve this outcome by judiciously specifying the distribution of the participation costs. The main thrust of the proposition, however, concerns the expression for the risky-asset prices.

Proposition 5 *There exist an interval $[\Delta_l, \Delta_u]$, a (non-trivial) distribution function $\Psi(\cdot)$ on $[\Delta_l, \Delta_u]$, and a cost function $F(\cdot; \Delta_t) : [\Delta_l, \Delta_u] \rightarrow \mathbb{R}^+$ such that, if Δ_t is drawn in an i.i.d. fashion from Ψ , then*

- (i) *investors optimally choose $\Delta = \Delta_t$, thus incurring cost $F(\Delta_t; \Delta_t)$;*
- (ii) *the risk-free rate is constant over time and given as the unique positive solution to*

$$1 = \beta(1+r) E \left[e^{\frac{\gamma^2}{2} \left(\frac{r}{1+r-\rho} \right)^2 (1-\rho)^2 \omega(\Delta)} \right], \quad (32)$$

- where the expectation is taken over the distribution of Δ ;
- (iii) *the risky-asset prices equal*

$$P_{t,j}(\Delta_t, D_{t,j}) = \phi(D_{t,j} - 1) + \frac{1}{r} - \Phi_1 \omega(\Delta_t) - \Phi_0 \quad (33)$$

with $\phi \equiv \frac{\rho}{1+r-\rho}$ and

$$\Phi_1 = \gamma \frac{r(1-\rho)^2}{(1+r-\rho)^2} \quad (34)$$

$$\Phi_0 = \frac{\Phi_1}{r} \frac{E \left[e^{\frac{\gamma^2}{2} \left(\frac{r}{1+r-\rho} \right)^2 (1-\rho)^2 \omega(\Delta)} \omega(\Delta) \right]}{E \left[e^{\frac{\gamma^2}{2} \left(\frac{r}{1+r-\rho} \right)^2 (1-\rho)^2 \omega(\Delta)} \right]} > 0. \quad (35)$$

Furthermore, investors' optimal portfolios of risky assets are given by (11).

Equation (33) decomposes the price of a security into three components. As in all CARA models, one of these components equals the expected discounted value of future dividends, $\phi(D_{t,j} - 1) + r^{-1}$. The other two capture the risk premium. The term $\Phi_1 \omega(\Delta_t)$ is the risk premium associated with the realization of time- $t + 1$ dividend uncertainty, to which each investor is exposed according to

²¹Alternatively, in the interest of simplicity, we could fix the interest exogenously to the model and let aggregate lending adjust accordingly.

the breadth Δ_t of her time- t portfolio. Finally, Φ_0 equals the sum of the expected discounted value of risk premia due to future realizations of dividend innovations and Δ_t .

We emphasize that the risk premium decreases with Δ_t and is common for all securities. Alternatively phrased, increases in capital movements across locations are correlated with higher prices for all risky securities (and hence lower expected excess returns). Importantly, these movements in the prices of risky securities are uncorrelated with movements in aggregate output or the interest rate, which are both constant by construction.

A further immediate implication of equation (33) is that the presence of repeated shocks to participation costs introduces correlation in security prices that exceeds that of their dividends. Indeed, taking two securities j and k , and noting that $Var(D_{t,j}) = Var(D_{t,k})$, we can use equation (33) to compute

$$corr(P_{t,j}, P_{t,i}) = \frac{Var(\Phi_1\omega(\Delta_t)) + \phi^2 cov(D_{t,j}, D_{t,k})}{Var(\Phi_1\omega(\Delta_t)) + \phi^2 Var(D_{t,j})} > \frac{cov(D_{t,j}, D_{t,k})}{Var(D_{t,j})} = corr(D_{t,j}, D_{t,i}).$$

The intuition is that movements in market integration cause common movements in the pricing of risk which make prices more correlated (and volatile) than the underlying dividends.

We collect some basic properties of the price due to the randomness in Δ_t in the following proposition.

- Proposition 6** (i) $P_{t,i}$ increases with Δ_t , and therefore $corr(P_{t,i}, \Delta_t) > 0$;
(ii) $E_t[P_{i,t+1} - (1+r)P_{i,t}]$ decreases with Δ_t ;
(iii) $corr(P_{t,i}, P_{t,j}) > corr(D_{t,i}, D_{t,j})$;
(iv) Φ_0 is higher (and hence the unconditional expected price is lower) than the one obtaining for Δ_t constant and equal to $E[\Delta]$.

A.2 Multiple arcs on the circle

The baseline model assumes that investors participate in markets spanning a single arc of length Δ around their “home” location. Extending the results to the general case where investors can choose to participate on multiple, disconnected arcs (as illustrated on the right-most graph of Figure 4) is straightforward and involves essentially no new insights. In this section we briefly sketch how to extend the results of the baseline model to this case and we show that allowing for this extra generality introduces an additional source of non-concavity into an investor’s optimization problem.

To start, we introduce the function

$$v(I) = \min_{N_i, \vec{a}_i, \vec{\Delta}_i, G_j^{(i)}} Var \left(\int_0^1 D_j dG_j^{(i)} \right)$$

$$s.t. \quad I \geq F \left(\vec{a}_i, \sum_{n=1}^{N_i} \Delta_{i,n} \right).$$

In words, the function $v(I)$ is the minimal variance, per share purchased, of the portfolio payoff that can be obtained by an investor who is willing to spend an amount I on participation costs. Proceeding similarly to Section 4.2 under the assumption that $P_j = P$ for all j , the facts that U is exponential and all D_j are normally distributed imply that maximizing utility over the choice of

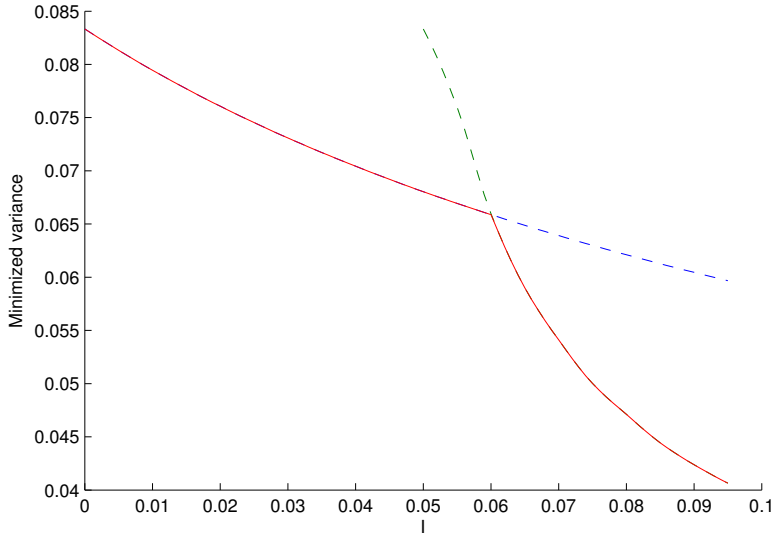


Figure 12: Numerical example to illustrate that $v(I)$ is non-convex. The figure depicts two (dotted) lines and the minimum of the two lines (solid line). The first dotted line starts at $I = 0$ and depicts the minimal variance that can be attained when participation costs are equal to I and the investor chooses to participate only on a single arc centered at her home location. The second dotted line starts at $I = 0.05$, i.e. at the minimum expenditure required to invest in two distinct arcs. This second dotted line depicts the minimal variance that can be attained when participation costs are equal to I and the investor can participate on two separate arcs with locations and lengths chosen so as to minimize variance. The function $v(I)$ (the minimum of the two dotted lines) is given by the solid line. For this example we chose $\sigma = 1$, $g(x) = 0.1 \times ((1-x)^{-6} - 1)$, $f(y) = 0.05 + 0.005 \times ((\frac{1}{2} - y)^{-2} - \frac{1}{0.25})$. (For $I > 0.1$ the function $v(I)$ would in general exhibit further kinks at the critical values I_N , $N \geq 2$, where the investor is indifferent between N and $N + 1$ distinct arcs.)

N_i , $\{a_{i,1}; \dots; a_{i,N_i}\}$, $\{\Delta_{i,1}; \dots; \Delta_{i,N_i}\}$, $G_j^{(i)}$, and w_i^f is equivalent to solving²²

$$\max_{w_i^f, I} Pw_i^f + (1 - w_i^f) \int_0^1 E[D_j] dG_j^{(i)} - \frac{\gamma}{2} (1 - w_i^f)^2 v(I) - I. \quad (36)$$

²²To ensure that the optimization problem (36) has a solution it is convenient either to impose a collateral constraint such as (22) or to introduce some (potentially small but positive) aggregate risk in dividends. Either of these assumptions coupled with the additional assumption $\lim_{\Delta \rightarrow 1} g(\Delta) = \infty$ suffices to ensure the existence of a solution to (36). Alternatively, one can ensure that (36) has an interior solution by requiring that, upon plugging in the optimal value of w_i^f , the maximand in (37) tends to negative infinity as I goes to infinity. Given the lower bound on P provided by the autarky equilibrium, it suffices that $\lim_{I \rightarrow \infty} \left(\frac{\sigma^2}{12}\right)^2 \frac{\gamma}{v(I)} - I = -\infty$. This condition may be harder to verify than Assumption 1, since it is not readily expressed in terms of primitive parameters.

Given that $E[D_j] = 1$, equation (36) can be rewritten as

$$V = \max_{I, w_i^f} Pw_i^f + (1 - w_i^f) - \frac{\gamma}{2} (1 - w_i^f)^2 v(I) - I. \quad (37)$$

In the baseline version of the model (Section 4.2), where the investor chooses to invest in a single arc around her home location, v is a convex function of the total cost I .²³ In the general case where investors' portfolios are invested on disconnected arcs, the function $v(I)$ is in general non-convex with kinks at the expenditure levels I_n where it becomes optimal to invest in $n + 1$ rather than n distinct arcs. Figure 12 provides an illustration. This non-convexity of $v(I)$, which may arise when (and only when) investors participate in markets located on multiple distinct arcs, constitutes an additional reason for the maximization problem (37) to be non-concave. This reason is distinct from the non-concavity arising from the interaction between leverage and participation decisions that we identify in Section 4.3, and strengthens the conclusion that a symmetric equilibrium may not exist.

B Proofs

Proof of Lemma 1. Property 2 follows immediately from integrating (2). To show property 3, note that, for any $i \in (0, 1)$, $\lim_{d(i,j) \rightarrow 0} D_j = \lim_{j \rightarrow i} D_j = D_i$ a.s. by the continuity of the Brownian motion. Continuity at 0 follows from the fact that $B_0 = B_1$.

We turn now to property 1. Since $E(B_i) = 0$ for all $i \in [0, 1]$, $E(D_j) = 1$. To compute $cov(D_i, D_j)$ we start by noting that $cov(B_s, B_t) = E(B_s B_t) = s(1 - t)$ for $s \leq t$. Therefore, for any $t \in [0, 1]$,

$$\begin{aligned} \int_0^1 E(B_t B_u) du &= \int_0^t u(1 - t) du + \int_t^1 t(1 - u) du \\ &= \frac{1}{2}(1 - t)t^2 + \frac{1}{2}(1 - t)^2 t = \frac{t(1 - t)}{2}. \end{aligned} \quad (38)$$

Accordingly,

$$\begin{aligned} Var\left(\int_0^1 B_u du\right) &= E\left[\left(\int_0^1 B_u du\right)^2\right] = E\left[\left(\int_0^1 B_u du\right)\left(\int_0^1 B_t dt\right)\right] \\ &= \int_0^1 \left(\int_0^1 E(B_u B_t) du\right) dt = \int_0^1 \frac{t(1 - t)}{2} dt = \frac{1}{12}, \end{aligned} \quad (39)$$

where the second line of (39) follows from Fubini's Theorem and (38). Combining (39) and (38)

²³To see this, note that $v'(I) = \frac{\omega'(\Delta)}{\kappa g'(\Delta)}$, where $\Delta(I) = g^{-1}\left(\frac{I}{\kappa}\right)$. Differentiating again gives $v''(I) = \frac{1}{\kappa} \frac{\omega''(\Delta)g'(\Delta) - \omega'(\Delta)g''(\Delta)}{(g'(\Delta))^2} \Delta'(I) > 0$.

gives

$$\begin{aligned} \frac{1}{\sigma^2} \text{Var}(D_t) &= \text{Var}(B_t) + \text{Var}\left(\int_0^1 B_u du\right) - 2\text{cov}\left(B_t, \int_0^1 B_u du\right) \\ &= t(1-t) + \frac{1}{12} - 2 \int_0^1 E(B_t B_u) du = \frac{1}{12}. \end{aligned} \quad (40)$$

This calculation finishes the proof of property 1. For property 4, take any $s \leq t$ and use (38) and (39) to obtain

$$\begin{aligned} \frac{\text{cov}(D_s, D_t)}{\sigma^2} &= \text{cov}\left(B_s - \int_0^1 B_u du, B_t - \int_0^1 B_u du\right) \\ &= E(B_s B_t) - E\left(B_s \int_0^1 B_u\right) du - E\left(B_t \int_0^1 B_u\right) du + \frac{1}{12} \\ &= s(1-t) - \frac{s(1-s)}{2} - \frac{t(1-t)}{2} + \frac{1}{12} \\ &= \frac{(s-t)(1+s-t)}{2} + \frac{1}{12}. \end{aligned} \quad (41)$$

This establishes property 4. ■

Proof of Proposition 1. We start by establishing the following lemma.

Lemma 3 *The (bounded-variation) function L with $L_{-\frac{\Delta}{2}-} = 0$ and $L_{\frac{\Delta}{2}} = 1$ that minimizes $\text{Var}\left(\int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} D_j dL_j\right)$ is given by (11). Moreover, the minimal variance is equal to $\omega(\Delta)$.*

Proof of Lemma 3. To simplify notation, we prove a “shifted” version of the lemma, namely finding the minimal-variance portfolio on $[0, \Delta]$ rather than $[-\frac{\Delta}{2}, \frac{\Delta}{2}]$. The two versions are clearly equivalent, since covariances depend only on the distances between locations, rather than the locations themselves.

We start by defining $q(d) = \frac{1}{12} - \frac{d(1-d)}{2}$ and therefore $q'(d) = -\frac{1}{2} + d$. In light of (3), $q(d) = \frac{1}{\sigma^2} \text{cov}(D_i, D_j)$ whenever $d(i, j) = d$. If $L_u = \int_{0-}^u dL_u$ is a variance-minimizing portfolio of risky assets, it must be the case that the covariance between any gross return $R_s = \frac{D_s}{P}$ for $s \in [0, \Delta]$ and the portfolio $\int_{0-}^{\Delta} R_u dL_u = \int_{0-}^{\Delta} \frac{D_u}{P} dL_u$ is independent of s . Thus, the quantity

$$\begin{aligned} \frac{1}{\sigma^2} \text{cov}\left(D_s, \int_{0-}^{\Delta} D_u dL_u\right) &= \frac{1}{\sigma^2} \left[\int_{0-}^s \text{cov}(D_s, D_u) dL_u + \int_s^{\Delta} \text{cov}(D_s, D_u) dL_u \right] = \\ &= \int_{0-}^s q(s-u) dL_u + \int_s^{\Delta} q(u-s) dL_u \end{aligned} \quad (42)$$

is independent of s . Letting $\tilde{L}(s) = 1 - L(s)$ and integrating by parts we obtain

$$\int_{0-}^s q(s-u) dL_u = L(s)q(0) - L(0^-)q(s) + \int_{0-}^s L_u q'(s-u) du \quad (43)$$

$$\int_s^{\Delta} q(u-s) dL_u = \tilde{L}(s)q(0) - \tilde{L}(\Delta)q(\Delta-s) + \int_s^{\Delta} \tilde{L}_u q'(u-s) du. \quad (44)$$

Using (43) and (44) inside (42) and recognizing that $q(0) = \frac{1}{12}$, $L(0^-) = 0$, and $\tilde{L}(\Delta) = 0$, we obtain that (42) equals

$$Q(s) \equiv \frac{1}{12} + \int_{0^-}^s L_u q'(s-u) du + \int_s^\Delta \tilde{L}_u q'(u-s) du. \quad (45)$$

This expression is independent of $s \in [0, \Delta]$ if and only if $Q'(s) = 0$. Differentiating (45) and setting the resulting expression to zero yields

$$\begin{aligned} Q'(s) &= \int_{0^-}^s L_u q''(s-u) du - \int_s^\Delta \tilde{L}_u q''(u-s) du + L_s q'(0) - \tilde{L}_s q'(0) \\ &= \int_{0^-}^\Delta L_u du - \Delta + s - L_s + \frac{1}{2} = 0, \end{aligned} \quad (46)$$

where we used $q'(0) = -\frac{1}{2}$, $q'' = 1$, and $\tilde{L}(s) = 1 - L(s)$. Since (46) needs to hold for all $s \in [0, \Delta]$, it must be the case that $L_s = A + s$ for an appropriate constant A . To determine A , we substitute $L_s = A + s$ into (46) and solve for A to obtain

$$A = \frac{1 - \Delta}{2}.$$

It is immediate that the standardized portfolio corresponding to the solution L we computed is L^* of (11).

Using the variance-minimizing portfolio inside (45), implies after several simplifications, that $Q = \frac{1}{12}(1 - \Delta)^3$ and hence $\text{cov}\left(D_s, \int_{0^-}^\Delta D_u dL_u\right) = Q\sigma^2 = \omega(\Delta)$. Accordingly,

$$\begin{aligned} \text{Var}\left(\int_{0^-}^\Delta D_u dL_u\right) &= \text{cov}\left(\int_{0^-}^\Delta D_s dL_s, \int_{0^-}^\Delta D_u dL_u\right) = \int_{0^-}^\Delta \text{cov}\left(D_s, \int_{0^-}^\Delta D_u dL_u\right) dL_s \\ &= \omega(\Delta) \int_{0^-}^\Delta dL_s = \omega(\Delta). \end{aligned}$$

■

With Lemma 3 in hand it is possible to confirm that the allocations and prices of Proposition 1 constitute a symmetric equilibrium — assuming that one exists. We already argued that all agents choose the same standardized portfolio (as agent $\frac{\Delta}{2}$). Furthermore, since in a symmetric equilibrium all agents must hold the same allocation of bonds, clearing of the bond market requires $w_i^f = 0$ for all i . By equation (17), $w_i^f = 0$ is supported as an optimal choice for an investor only if $P_i = P$ is given by (14). Similarly, in light of (18), equation (13) is a necessary optimality condition for the interval Δ^* . Since the values of P and Δ^* implied by (14) and (13) are unique, they are necessarily the equilibrium values of P and Δ^* that characterize a symmetric equilibrium.

Existence of a symmetric equilibrium implies that $w_i^f = 0$ is optimal, and so are the choices Δ^* and $G_{i+j}^{(i)} = L_j^*$ given prices $P_i = P$. It remains to show that markets clear. We already addressed bond-market clearing. To see that the stock markets clear, we start by noting that, since $P_i = W_{0,i} = P$ for all i , the market clearing condition amounts to $\int_{i \in [0,1]} dG_j^{(i)} = 1$. We have $\int_{i \in [0,1]} dG_j^{(i)} = \int_{i \in [0,1]} dL_{j-i}^* = \int_{j \in [0,1]} dL_j^* = 1$. ■

Proof of Proposition 2. Let $w^*(P)$ denote the set of optimal w_i^f solving the maximization

problem (16) when the price in all markets is P . We first note that the assumption that no symmetric equilibrium exists implies that there exists no P such that $0 \in w^*(P)$. (If such a P existed, then we could simply repeat the arguments of Proposition 1 to establish the existence of a symmetric equilibrium with price $P_i = P$, and interval choice $\Delta_i = \Delta^*(P)$).

We next show that since there exists no P such that $0 \in w^*(P)$, it follows that $w^*(P)$ cannot be single-valued for all P . We argue by contradiction. Suppose to the contrary that $w^*(P)$ is single-valued. Since the theorem of the maximum implies that $w^*(P)$ is an upper-hemicontinuous correspondence, it follows that $w^*(P)$ is actually a continuous function. Inspection of (16) shows that $w^*(1) = 1$. Moreover, as $P \rightarrow -\infty$, the optimal solution to (16) becomes negative: $w^*(-\infty) < 0$. Then an application of the intermediate value theorem gives the existence of P such that $w^*(P) = 0$, a contradiction.

Combining the facts that a) there exists no P such that $0 \in w^*(P)$, b) $w^*(P)$ is multi-valued for at least one value of P , and c) $w^*(P)$ is upper-hemicontinuous, implies that there exists at least one P such that $\{w_1, w_2\} \in w^*(P)$ with $w_1 > 0$ and $w_2 < 0$. An implication of the necessary first-order condition for the optimality of the interval choice $\Delta^*(P)$ is that $\Delta^*(P)$ is also multi-valued with $\Delta_1 < \Delta_2$. Furthermore, since prices in all locations are equal, the (standardized) optimal portfolio of an agent choosing Δ_k is the variance-minimizing portfolio of Proposition 1, denoted $L^{*,k}$.

From this point onwards, an equilibrium can be constructed as follows. By definition, the tuples $\{\Delta_1, w_1, dL^{*,1}\}$ and $\{\Delta_2, w_2, dL^{*,2}\}$ are optimal. Hence it only remains to confirm that asset markets clear. Define $\pi \equiv -\frac{w_2}{w_1 - w_2} \in (0, 1)$. By construction, $\pi w_1 + (1 - \pi)w_2 = 0$ and, therefore, if in every location π agents choose $\{\Delta_1, w_1, dL^{*,1}\}$ and the remaining fraction $(1 - \pi)$ choose $\{\Delta_2, w_2, dL^{*,2}\}$, then the bond market clears by construction. To see that the stock markets clear, we start by noting that, since $P_i = W_{0,i} = P$, the market clearing condition for stock i amounts to

$$\pi \int_{[0,1]} dL_{j-i}^{*,1} + (1 - \pi) \int_{[0,1]} dL_{j-i}^{*,2} = 1,$$

which holds because $L_j^{*,k}$ for $k \in \{1, 2\}$ is a measure on the circle. ■

Remark 1 *The existence proof of an asymmetric equilibrium (when a symmetric equilibrium fails to exist) obtains also in the presence of the leverage constraint (22) that we introduce in Section 5.*

Proof of Proposition 3. Conjecture first that in equilibrium $P_j = P$ for all j and let $\pi \in [0, 1]$ denote the fraction of funds invested in the local market. Assuming that a given investor chooses $N = 2$ (that is, chooses to invest in her own location and another location at distance d), equation (3) allows the computation of the minimal portfolio variance:

$$\begin{aligned} \widehat{w}(d) &= \sigma^2 \min_{\pi} \left\{ \left(\pi^2 + (1 - \pi)^2 \right) \frac{1}{12} + 2\pi(1 - \pi) \left(\frac{1}{12} - \frac{d(1 - d)}{2} \right) \right\} \\ &= \sigma^2 \left(\frac{1}{12} - \frac{1}{4}d(1 - d) \right). \end{aligned} \tag{47}$$

The optimal distance d for an investor choosing $N = 2$ satisfies a first-order condition similar to equation (18), namely

$$-\frac{\gamma}{2} (1 - w^f)^2 \frac{\widehat{w}'(d)}{\widehat{w}^2(d)} = \kappa f'(d). \tag{48}$$

Since $\widehat{w}'(d) = 0$ when and only when $d = \frac{1}{2}$, and $f'(d) = 0$, it follows that $d = \frac{1}{2}$ is optimal for an investor choosing $N = 2$. Hence the minimal portfolio variance of an investor choosing $N = 1$ is equal to $\widehat{w}(0) = \frac{\sigma^2}{12}$, while the minimal portfolio variance for an investor choosing $N = 2$ is $\widehat{w}(\frac{1}{2}) = \frac{\sigma^2}{48}$. Assuming that the equilibrium is of the asymmetric, location-invariant type, we can use equation (16) to express the indifference between the choices $N = 1$, respectively $N = 2$ and $d = \frac{1}{2}$, as

$$Pw_1^f + (1 - w_1^f) - \frac{\gamma}{2} (1 - w_1^f)^2 \widehat{w}(0) = Pw_2^f + (1 - w_2^f) - \frac{\gamma}{2} (1 - w_2^f)^2 \widehat{w}\left(\frac{1}{2}\right) - \kappa f_0. \quad (49)$$

Using the first-order conditions for leverage

$$1 - P = \gamma (1 - w_2^f) \widehat{w}\left(\frac{1}{2}\right) = \gamma (1 - w_1^f) \widehat{w}(0) \quad (50)$$

inside (49) yields — after some simplifications — the equilibrium price (23).

To verify that the postulated equilibrium is indeed an equilibrium, we proceed as in the proof of Proposition 2. For $P(\kappa)$ to be an equilibrium price in all locations, it must also be case that $1 - w_1^f \leq 1 \leq 1 - w_2^f$, so that setting $\pi = \frac{w_2^f}{w_2^f - w_1^f} > 0$ ensures market clearing (of bond markets and all risky asset markets). In light of (50), the requirement $1 - w_1^f \leq 1 \leq 1 - w_2^f$ is equivalent to $P \in [1 - \gamma \widehat{w}(0), 1 - \gamma \widehat{w}(\frac{1}{2})]$. This requirement is satisfied as long as $\kappa \in (\kappa_1, \kappa_2)$.

Finally, (24) follows from (49), upon applying the implicit function theorem and utilizing the envelope theorem.

■

Proof of Lemma 2. In an asymmetric equilibrium the value functions of the agents who pursue the strategy of setting $N = 1$ (respectively $N = 2$) are given by

$$V_1 = \max_{w_1^f} Pw_1^f + (1 - w_1^f) - \frac{\gamma}{2} (1 - w_1^f)^2 \widehat{w}(0), \quad (51)$$

$$V_2 = \max_{w_2^f} Pw_2^f + (1 - w_2^f) - \frac{\gamma}{2} (1 - w_2^f)^2 \widehat{w}\left(\frac{1}{2}\right) - \kappa f_0, \quad (52)$$

where the second maximization is subject to the constraint (22), and $\widehat{w}(d)$ is given by equation (47). Indifference requires that $V_1 = V_2$, and hence $\frac{dV_1}{d\kappa} - \frac{dV_2}{d\kappa} = 0$.

Attaching a Lagrange multiplier $\lambda \geq 0$ to the constraint (25), solving the resulting maximization problem, and utilizing the envelope theorem implies

$$\frac{dV_1}{d\kappa} - \frac{dV_2}{d\kappa} = \left(w_1^f - w_2^f - \lambda \left[1 - \chi (1 - w_2^f) \right] \right) \frac{dP}{d\kappa} + (1 + \lambda) f_0, \quad (53)$$

with $\lambda = \frac{1 - P - \gamma (1 - w_2^f) \widehat{w}(\frac{1}{2})}{\chi P} \geq 0$. Setting the right-hand side of (53), equal to zero and re-arranging gives (27).

■

Lemma 4 Consider an investor located at $i \notin [-\frac{k}{2}, \frac{k}{2}]$, and therefore investing in markets $[i - \frac{\Delta}{2}, i + \frac{\Delta}{2}]$. Suppose that $P(x)$ is continuously differentiable everywhere on $[i - \frac{\Delta}{2}, i + \frac{\Delta}{2}]$. With

$dX_l^{(i)}$ the number of shares purchased on the account of an investor at i in market l and $j \equiv i - \frac{\bar{\Delta}}{2}$,

$$X_{j+\bar{\Delta}}^{(i)} = \frac{1}{\gamma\omega(\bar{\Delta})} \left[1 - \frac{1-\bar{\Delta}}{2} (P_j + P_{j+\bar{\Delta}}) - \int_j^{j+\bar{\Delta}} P_u du \right]. \quad (54)$$

Furthermore, the function X is given by

$$X_{j+l}^{(i)} = \frac{P'_{j+l}}{\gamma\sigma^2} + X_{j+\bar{\Delta}}^{(i)} \frac{1-\bar{\Delta}+2l}{2} + \frac{1}{1-\bar{\Delta}} \frac{P_{j+\bar{\Delta}} - P_j}{\gamma\sigma^2}. \quad (55)$$

If an investor is located at $i \in [-\frac{k}{2}, \frac{k}{2}]$ and only invests in market i then the respective demand for risky asset i is given by

$$\widehat{X}_i^{(i)} = \frac{1}{\gamma\omega(0)} (1 - P_i). \quad (56)$$

Proof of Lemma 4. Notice that optimization problem of agent i is equivalent to

$$\max_X P_i + \int_{j^-}^{j+\bar{\Delta}} (1 - P_u) dX_u - \frac{\gamma}{2} \text{Var} \left(\int_{j^-}^{j+\bar{\Delta}} D_u dX_u \right) \quad (57)$$

Thus, the first-order condition requires that

$$\gamma \text{cov} \left(D_s, \int_{j^-}^{j+\bar{\Delta}} D_u dX_u \right) = 1 - P_s \quad (58)$$

for all $s \in [j, j + \bar{\Delta}]$. Letting $q(d)$ be defined as in Lemma 3 we can rewrite (58) as

$$\int_{j^-}^s q(s-u) dX_u + \int_s^{j+\bar{\Delta}} q(u-s) dX_u = \frac{1 - P_s}{\gamma\sigma^2}. \quad (59)$$

Let $\tilde{X}(s) = X(j + \bar{\Delta}) - X(s)$ and integrating by parts we obtain

$$\int_{j^-}^s q(s-u) dX_u = X(s)q(0) - X(j^-)q(s) + \int_j^s X_u q'(s-u) du \quad (60)$$

$$\int_s^{j+\bar{\Delta}} q(u-s) dX_u = \tilde{X}(s)q(0) - \tilde{X}(\bar{\Delta})q(\Delta-s) + \int_s^{j+\bar{\Delta}} \tilde{X}_u q'(u-s) du \quad (61)$$

Substituting (60) and (61) into (59), recognizing that $q(0) = \frac{1}{12}$, $X(j^-) = 0$, and $\tilde{X}(j + \bar{\Delta}) = 0$, we obtain

$$\frac{1}{12} X(j + \bar{\Delta}) + \int_j^s X_u q'(s-u) du + \int_s^{j+\bar{\Delta}} \tilde{X}_u q'(u-s) du = \frac{1 - P_s}{\gamma\sigma^2}. \quad (62)$$

Since this relation must hold for all s , we may differentiate both sides of (62) to obtain

$$\int_j^s X_u q''(s-u) du - \int_s^{j+\bar{\Delta}} \tilde{X}_u q''(u-s) du + X_s q'(0) - \tilde{X}_s q'(0) = -\frac{P'_s}{\gamma\sigma^2}. \quad (63)$$

This equation holds for all $s \in (j, j + \bar{\Delta})$. Noting that $q'' = 1$, $q'(0) = -\frac{1}{2}$, $\tilde{X}(s) = X(j + \bar{\Delta}) - X(s)$, and using (63) to solve for X_s yields

$$X_s = \int_j^{j+\bar{\Delta}} X_u du + \left(s - j + \frac{1}{2} - \bar{\Delta}\right) X(j + \bar{\Delta}) + \frac{P'_s}{\gamma\sigma^2}. \quad (64)$$

Integrating (64) from j to $j + \bar{\Delta}$ and solving for $\int_j^{j+\bar{\Delta}} X_u du$ leads to

$$\int_j^{j+\bar{\Delta}} X_u du = \frac{1}{1 - \bar{\Delta}} \left[X(j + \bar{\Delta}) \bar{\Delta} \left(\frac{1 - \bar{\Delta}}{2}\right) + \frac{P(j + \bar{\Delta}) - P(j)}{\gamma\sigma^2} \right], \quad (65)$$

so that

$$X_s = \frac{1}{1 - \bar{\Delta}} \left[X(j + \bar{\Delta}) \bar{\Delta} \left(\frac{1 - \bar{\Delta}}{2}\right) + \frac{P(j + \bar{\Delta}) - P(j)}{\gamma\sigma^2} \right] + \left(s - j + \frac{1}{2} - \bar{\Delta}\right) X(j + \bar{\Delta}) + \frac{P'_s}{\gamma\sigma^2}. \quad (66)$$

Evaluating (62) at $s = j + \bar{\Delta}$, and noting that $q'(s) = -\frac{1}{2} + s$ leads to

$$\frac{1}{12} X(j + \bar{\Delta}) + \int_j^{j+\bar{\Delta}} X_u \left[-\frac{1}{2} + (\bar{\Delta} - u)\right] du = \frac{1 - P_{j+\bar{\Delta}}}{\gamma\sigma^2}. \quad (67)$$

An implication of (64) is that $X_u = X_j + \frac{P'_u - P'_j}{\gamma\sigma^2} + X(j + \bar{\Delta})u$. Using this expression for X_u inside (67), carrying out the requisite integrations and using integration by parts to express $\int_j^{j+\bar{\Delta}} \left(\frac{P'_u}{\gamma\sigma^2}\right) u du = \frac{P_{j+\bar{\Delta}}}{\gamma\sigma^2}(j + \bar{\Delta}) - \frac{P_j}{\gamma\sigma^2}j - \int_j^{j+\bar{\Delta}} \frac{P_u}{\gamma\sigma^2} du$, leads (after some simplifications) to

$$X(j + \bar{\Delta}) \left(\frac{1}{12} + \frac{\bar{\Delta}^3}{6} - \frac{\bar{\Delta}^2}{4}\right) - \frac{\bar{\Delta}(1 - \bar{\Delta})}{2} \left(X_j - \frac{P'_j}{\gamma\sigma^2}\right) - \frac{P_{j+\bar{\Delta}} - P_j}{2\gamma\sigma^2} + \int_{j^-}^{j+\bar{\Delta}} \frac{P_u - P_j}{\gamma\sigma^2} du = \frac{1 - P_{\bar{\Delta}}}{\gamma\sigma^2}. \quad (68)$$

Finally, evaluating (64) at j gives

$$\left(X_j - \frac{P'_j}{\gamma\sigma^2}\right) = \int_j^{j+\bar{\Delta}} X_u du + \left(\frac{1}{2} - \bar{\Delta}\right) X(j + \bar{\Delta}). \quad (69)$$

Equations (65), (68), and (69) are three linear equations in three unknowns. Solving for $X(j + \bar{\Delta})$ and using the definition of $\omega(\bar{\Delta})$ leads to (54). Equation (66) simplifies to (55). Finally, (56) is a direct consequence of (58) when $\bar{\Delta} = 0$. ■

Proof of Proposition 4. For any $j \in (\frac{k}{2}, \frac{1}{2}]$ and $l \in (-\frac{\bar{\Delta}}{2}, \frac{\bar{\Delta}}{2})$, we have from Lemma 4:

$$\begin{aligned} X_{j-\frac{\bar{\Delta}}{2}}^{(j)} &= \frac{P'_{j-\frac{\bar{\Delta}}{2}}}{\gamma\sigma^2} + \frac{P_{j-\frac{\bar{\Delta}}{2}} - P_{j+\frac{\bar{\Delta}}{2}}}{\gamma\sigma^2(1-\Delta)} + \frac{1-\Delta}{2} X_{j+\frac{\bar{\Delta}}{2}}^{(j)} \\ dX_{j+l}^{(j)} &= \left(\frac{P''_{j+l}}{\gamma\sigma^2} + X_{j+\frac{\bar{\Delta}}{2}}^{(j)} \right) dl \\ X_{j+\frac{\bar{\Delta}}{2}}^{(j)} - X_{(j+\frac{\bar{\Delta}}{2})^-}^{(j)} &= -\frac{P'_{j+\frac{\bar{\Delta}}{2}}}{\gamma\sigma^2} - \frac{P_{j-\frac{\bar{\Delta}}{2}} - P_{j+\frac{\bar{\Delta}}{2}}}{\gamma\sigma^2(1-\Delta)} + \frac{1-\Delta}{2} X_{j+\frac{\bar{\Delta}}{2}}^{(j)}. \end{aligned}$$

Specialize the first equation to $j = \frac{1}{2} + \frac{\bar{\Delta}}{2}$, the second to $j = \frac{1}{2} - l$ for all $l \in (-\frac{\bar{\Delta}}{2}, \frac{\bar{\Delta}}{2})$, and the third to $j = \frac{1}{2} - \frac{\bar{\Delta}}{2}$ and aggregate to obtain the total demand for asset $\frac{1}{2}$:

$$1 = \frac{1-\Delta}{2} X_{\frac{1}{2}+\bar{\Delta}}^{(\frac{1}{2}+\frac{\bar{\Delta}}{2})} + \frac{1-\Delta}{2} X_{\frac{1}{2}}^{(\frac{1}{2}-\frac{\bar{\Delta}}{2})} + \int_{-\frac{\bar{\Delta}}{2}}^{\frac{\bar{\Delta}}{2}} X_{\frac{1}{2}-l+\frac{\bar{\Delta}}{2}}^{(\frac{1}{2}-l)} dl + \frac{P''_{\frac{1}{2}}}{\gamma\sigma^2} \Delta + \frac{P_{\frac{1}{2}-\bar{\Delta}} + P_{\frac{1}{2}+\bar{\Delta}} - 2P_{\frac{1}{2}}}{2\gamma\sigma^2(1-\Delta)}.$$

Suppose now that $P_j \geq 1 - \gamma\omega(\bar{\Delta})$ on $[\frac{1}{2} - \Delta, \frac{1}{2} + \Delta]$, with strict inequality on a positive measure set. It then follows from equation (54) that $X_{j+\frac{\bar{\Delta}}{2}}^{(j)} \leq 1$, so that

$$0 < \frac{P''_{\frac{1}{2}}}{\gamma\sigma^2} \Delta + \frac{P_{\frac{1}{2}-\bar{\Delta}} + P_{\frac{1}{2}+\bar{\Delta}} - 2P_{\frac{1}{2}}}{2\gamma\sigma^2(1-\Delta)}.$$

This inequality contradicts the assumption that P is maximized at $\frac{1}{2}$. ■

Proof of Proposition 5. We adopt a guess-and-verify approach. We start by noting that the beginning-of-period wealth of investor i at time $t+1$ is $W_{t+1,i} \equiv \int_0^1 (P_{t+1,j} + D_{t+1,j}) dX_{t,j}^{(i)} + X_{B,t}^{(i)}$. We then conjecture that, as long as

$$F(\Delta) = \frac{M}{\gamma} + \frac{\gamma}{2} \left(\frac{r}{1+r-\rho} \right)^2 (1-\rho)^2 \omega(\Delta_t) \quad (70)$$

for some $M > -\frac{\gamma^2}{2} \left(\frac{r}{1+r-\rho} \right)^2 (1-\rho)^2 \omega(\Delta_u)$, (ii) and (iii) obtain. We show at the end that the function F can be chosen to ensure (i).

We also conjecture and verify that investors' holdings of risky assets $X_{t,j}^{(i)}$ coincide with $G_{t,j}^{(i)}$ of Proposition 1, and that their bond holdings equal

$$X_{B,t}^{(i)} = W_{t,i} - (1+r)\bar{P}_{t,i} - r\Phi_t, \quad (71)$$

where $\bar{P}_{t,i} \equiv \int_0^1 P_{t,j} dX_{t,j}^{(i)}$ is the average price that investor i pays for her portfolio. Here, to simplify notation, we defined $\Phi_t \equiv \Phi_1 \omega(\Delta_t) + \Phi_0$.

We first ensure that with these postulates markets clear. Clearly, all risky markets clear, since the holdings of risky assets are the same as in Proposition 1. To show that bond markets clear, we proceed inductively. First we note that investors are endowed with no bonds at time

zero. Hence $\int_0^1 X_{B,-1}^{(i)} di = 0$ and therefore $\int_0^1 W_{0,i} di = \int_0^1 (P_{0,i} + D_{0,i}) di$. Next we postulate that $\int_0^1 X_{B,t-1}^{(i)} di = 0$, so that $\int_0^1 W_{t,i} di = \int_0^1 P_{t,i} di + \int_0^1 D_{t,j} dj$. Integrating our postulate (71) for $X_{B,t}^{(i)}$ across all investors, we obtain

$$\int_0^1 X_{B,t}^{(i)} di = \int_0^1 W_{t,i} di - (1+r) \int_0^1 \bar{P}_{t,i} di - r\Phi_t. \quad (72)$$

We next note that that (a) $\int_0^1 D_{t,j} dj = 1$, by construction of the dividend process; (b) $\int_0^1 W_{t,i} di = \int_0^1 P_{t,i} di + \int_0^1 D_{t,j} dj = r^{-1} - \Phi_t + 1$, using the induction hypothesis, (33), and (a); and (c) $\int_0^1 \bar{P}_{t,i} di = \int_0^1 P_{t,i} di = r^{-1} - \Phi_t$. Using these three facts, it follows immediately that the right-hand side of (72) is zero, so that the bond market clears.

If investors set their bond holdings according to (71), then their budget constraint implies a consumption of

$$c_{t,i} = W_{t,i} - \frac{1}{1+r} X_{B,t}^{(i)} - \bar{P}_{t,i} - F_t. \quad (73)$$

Using the definition of $W_{t,i}$ and market clearing condition for bond holdings inside (73), and integrating across i implies that the market for consumption goods clears: $\int_0^1 c_{t,i} di = 1 - F_t$.

Having established market clearing given the postulated policies and prices, we next turn to optimality. Equation (73) implies

$$\begin{aligned} c_{t+1,i} - c_{t,i} &= W_{t+1,i} - W_{t,i} - \frac{1}{1+r} (X_{B,t+1}^{(i)} - X_{B,t}^{(i)}) - (\bar{P}_{t+1,i} - \bar{P}_{t,i}) - (F_{t+1} - F_t) \\ &= \left(\frac{r}{1+r} \right) (W_{t+1,i} - W_{t,i}) + \frac{r}{1+r} (\Phi_{t+1} - \Phi_t) - (F_{t+1} - F_t), \end{aligned} \quad (74)$$

where the second line follows from (71). We next use the definition of $W_{t,i}$ and (71) to obtain

$$\begin{aligned} W_{t+1,i} - W_{t,i} &= \int_0^1 (P_{t+1,j} + D_{t+1,j}) dX_{t,j}^{(i)} + X_{B,t}^{(i)} - W_{t,i} \\ &= \int_0^1 (P_{t+1,j} + D_{t+1,j}) dX_{t,j}^{(i)} - (1+r) \bar{P}_{t,i} - r\Phi_t. \end{aligned} \quad (75)$$

Substituting (75) into (74) and using (33) and (71) leads to

$$c_{t+1,i} - c_{t,i} = \left(\frac{r}{1+r} \right) \left[(1+\phi) \int_0^1 D_{t+1,j} dX_{t,j}^{(i)} - (1+r) \phi \int_0^1 D_{t,j} dX_{t,j}^{(i)} - 1 \right] - (F_{t+1} - F_t). \quad (76)$$

Next use the fact $\bar{D}_{t+1,j} = \rho \bar{D}_{t,j} + (1-\rho) \bar{\varepsilon}_{t+1,j}$ along with $\phi = \frac{\rho}{1+r-\rho}$, $(1+\phi)\rho = (1+r)\phi$, and $(1+\phi)(1-\rho) = (1-r\phi)$ inside (76) to arrive at

$$c_{t+1,i} - c_{t,i} = \left(\frac{r}{1+r-\rho} \right) (1-\rho) \int_0^1 (\varepsilon_{t+1,j} - 1) dX_{t,j}^{(i)} - (F_{t+1} - F_t). \quad (77)$$

Having established (77), the dynamics of agent i 's consumption under our postulate, we next

turn attention to the Euler equations, starting with the bond Euler equation

$$1 = \beta (1 + r) E_t e^{-\gamma(c_{t+1,i} - c_{t,i})}. \quad (78)$$

Substituting (77) into (78) and noting that $\int_0^1 (\varepsilon_{t+1,j} - 1) dX_{t,j}^{(i)}$ is normally distributed with mean zero and variance $\omega(\Delta_t)$ gives

$$1 = \beta (1 + r) e^{\frac{\gamma^2}{2} \left(\frac{r}{1+r-\rho}\right)^2 (1-\rho)^2 \omega(\Delta_t) - \gamma F_t} E_t (e^{\gamma F_{t+1}}). \quad (79)$$

Now suppose that for any r and a given desired distribution $\Psi(\Delta)$ we set

$$F_t(\Delta_t; r) = \frac{M}{\gamma} + \frac{\gamma}{2} \left(\frac{r}{1+r-\rho}\right)^2 (1-\rho)^2 \omega(\Delta_t). \quad (80)$$

Then equation (79) can be written as (32). Since $(1+r) E e^{\frac{\gamma^2}{2} \left(\frac{r}{1+r-\rho}\right)^2 (1-\rho)^2 \omega(\Delta)}$ is equal to 1 when $r = 0$ and increases monotonically to infinity as r increases, it follows that there exists a unique positive r such that equation (32) holds. For that value of r , all investors' bond Euler equations are satisfied.

Finally, we need to determine Φ_t so as to ensure that the Euler equations for risky assets hold, i.e., that

$$P_{t,j} = \beta E_t \left[e^{-\gamma(c_{t+1,i} - c_{t,i})} (P_{t+1,j} + D_{t+1,j}) \right]. \quad (81)$$

To that end, we use (33) and (30) to express (81) as

$$\frac{1}{r} - \Phi_t + \phi(D_{t,j} - 1) = \beta E_t \left[e^{-\gamma(c_{t+1,i} - c_{t,i})} \left(\frac{1}{r} - \Phi_{t+1} + (1 + \phi) (\rho D_{t,j} + (1 - \rho) \varepsilon_{t+1,j}) - \phi \right) \right]. \quad (82)$$

We next note that

$$\beta E_t \left[e^{-\gamma(c_{t+1,i} - c_{t,i})} \right] (1 + \phi) \rho D_{t,j} = \frac{(1 + \phi) \rho}{1 + r} D_{t,j} = \phi D_{t,j} \quad (83)$$

using (78). Equation (83) simplifies (82) to

$$\begin{aligned} \frac{1}{r} - \Phi_t - \phi &= \beta E_t \left[e^{-\gamma(c_{t+1,i} - c_{t,i})} \left(\frac{1}{r} - \Phi_{t+1} + (1 + \phi) (1 - \rho) \varepsilon_{t+1,j} \right) \right] \\ &= \frac{1}{r(1+r)} - \beta E_t \left[e^{-\gamma(c_{t+1,i} - c_{t,i})} \Phi_{t+1} \right] + \frac{1}{1+r} (-\phi + (1 + \phi) (1 - \rho)) \\ &\quad + (1 + \phi) (1 - \rho) \beta E_t \left[e^{-\gamma(c_{t+1,i} - c_{t,i})} (\varepsilon_{t+1,j} - 1) \right]. \end{aligned} \quad (84)$$

Using (77), Stein's Lemma, the fact that $cov \left(\int_0^1 (\varepsilon_{t+1,j} - 1) dX_{t,j}^{(i)}, \varepsilon_{t+1,j} \right) = \omega(\Delta)$ (see Proposition 1), and (78) implies

$$\beta E_t \left[e^{-\gamma(c_{t+1,i} - c_{t,i})} \varepsilon_{t+1,j} \right] = \frac{1 - \gamma \frac{r}{1+r-\rho} (1 - \rho) \omega(\Delta_t)}{1 + r}. \quad (85)$$

Substituting (85) into (84) gives linear equations in Φ_0 and Φ_1 , solved by (34), respectively (35).

To complete the proof of the claim that Δ_t is chosen optimally, we provide an explicit example of a family of functions for $F_t(\Delta)$ that has the desired properties. To start, we compute the value function of an investor adopting the policies of Proposition 5. Equation (78) along with (77) imply that

$$\begin{aligned} V(W_{t,i}, \Delta_t) &= -\frac{1}{\gamma} \sum_{t \geq 0} \beta^t E_t [e^{-\gamma c_{t,i}}] = -\frac{1}{\gamma} e^{-\gamma c_{0,i}} \sum_{t \geq 1} \beta^t E_t \left[e^{-\gamma \sum_{m=0}^t (c_{t+1,i} - c_{t,i})} \right] \\ &= -\frac{1}{\gamma} e^{-\gamma c_{0,i}} \sum_{t \geq 0} (1+r)^{-t} = -\frac{1}{\gamma r} e^{-\gamma c_{0,i}}. \end{aligned}$$

In turn equations (71), (73), and (80) imply that

$$V(W_t, \Delta_t) = -\frac{1}{\gamma r} e^{-\frac{\gamma r}{1+r} W_{t,i} + z(\Delta_t)}, \quad (86)$$

where $z_t(\Delta_t) \equiv \frac{r\Phi_t}{1+r} - M + \frac{\gamma^2}{2} \left(\frac{r}{1+r-\rho} \right)^2 (1-\rho)^2 \omega(\Delta_t)$.

Next we suppose that we no longer impose that the investor choose $\Delta = \Delta_t$, (where Δ_t is the time- t random draw of Δ that we imposed in Proposition 5). Instead Δ is chosen optimally. However, prices are still given by $P_{t,j}(\Delta_t, D_{t,j})$ from equation (33). We will construct a function $\kappa_t g_t(\Delta)$ that renders the choice $\Delta = \Delta_t$ optimal at the total cost specified in (80).

Throughout we let $\bar{X}_t^{(i)}(\Delta; \Delta_t)$ denote the optimal number of total risky assets chosen by investor i , and assuming that that investor chooses Δ and prices are given by $P_{t,j}(\Delta_t, D_{t,j})$. For future reference, we note that by construction of the price function $P_{t,j}(\Delta_t, D_{t,j})$ it follows that $\bar{X}_t^{(i)}(\Delta_t; \Delta_t) = 1$. Using (86) the first order condition characterizing an optimal Δ is

$$F'_t(\Delta) = h(\Delta; \Delta_t),$$

where

$$h(\Delta; \Delta_t) = -\frac{1}{1+r} \frac{\gamma}{2} \left(\frac{r}{1+r-\rho} \right)^2 (1-\rho)^2 \left(\bar{X}_t^{(i)}(\Delta; \Delta_t) \right)^2 \omega'(\Delta). \quad (87)$$

Next we fix a value of Δ_t and we simplify notation by writing $h(\Delta)$ rather than $h(\Delta; \Delta_t)$. We also let $q(x)$ denote some continuous function with $q(0) = 1$ and $q(x) > 1$ for $x > 0$. Let $\eta \in [0, 1]$, take some positive (small) $\varepsilon < \frac{\Delta_t}{2}$, and consider the function

$$F'_t(\Delta) = \begin{cases} \frac{\Delta}{\varepsilon} \eta h(\varepsilon) & \text{for } \Delta \leq \varepsilon \\ \eta h(\Delta) & \text{for } \Delta \in (\varepsilon, \Delta_t - \varepsilon] \\ \eta h(\Delta_t - \varepsilon) \frac{\Delta_t - \Delta}{\varepsilon} + h(\Delta_t) \frac{\Delta - \Delta_t + \varepsilon}{\varepsilon} & \text{for } \Delta \in (\Delta_t - \varepsilon, \Delta_t] \\ h(\Delta) q(\Delta) & \text{for } \Delta > \Delta_t + \varepsilon. \end{cases} \quad (88)$$

By construction, $F'_t(0) = 0$ and $F'_t(\Delta)$ is continuous and increasing in Δ . More importantly, $F'_t(\Delta_t) = h(\Delta_t)$, and hence $\Delta = \Delta_t$ satisfies the necessary first order condition (87). Moreover, since $F'_t(\Delta) < (>) h(\Delta)$ for $\Delta < (>) \Delta_t$, it follows that $\Delta = \Delta_t$ is optimal for any $\varepsilon > 0$ and

$\eta \in [0, 1]$. Finally,

$$\lim_{\varepsilon \rightarrow 0} \int_0^{\Delta_t} F'_t(\Delta) = \eta \int_0^{\Delta_t} h(x) dx > 0. \quad (89)$$

Now suppose that nature draws $\Delta_t = \Delta_u > 0$. By choosing M that is sufficiently close to $-\frac{\gamma}{2} \left(\frac{r}{1+r-\rho}\right)^2 (1-\rho)^2 \omega(\underline{\Delta})$ it follows that

$$0 < \frac{M}{\gamma} + \frac{\gamma}{2} \left(\frac{r}{1+r-\rho}\right)^2 (1-\rho)^2 \omega(\Delta_u) < \int_0^{\Delta_u} h(x) dx. \quad (90)$$

Combining equations (89) and (90) it follows that for sufficiently small $\varepsilon > 0$ there exists some $\eta \in [0, 1]$ so that

$$\int_0^{\Delta_u} F'_t(x) dx = \frac{M}{\gamma} + \frac{\gamma}{2} \left(\frac{r}{1+r-\rho}\right)^2 (1-\rho)^2 \omega(\Delta_u) > 0. \quad (91)$$

Hence, when $\Delta_t = \Delta_u$ the cost function $\kappa_t g_t(\Delta)$ renders $\Delta = \Delta_u$, while also satisfying (80). The same argument implies that for any value of Δ_t that satisfies

$$0 < \frac{M}{\gamma} + \frac{\gamma}{2} \left(\frac{r}{1+r-\rho}\right)^2 (1-\rho)^2 \omega(\Delta_t) < \int_0^{\Delta_t} h(x) dx, \quad (92)$$

there exists $\eta \in [0, 1]$ and sufficiently small $\varepsilon > 0$ such that the optimal Δ coincides with Δ_t , and (80) holds. Continuity of $\omega(\Delta_t)$ and of $\int_0^{\Delta_t} h(x) dx$ in Δ_t implies that as long as $\underline{\Delta}$ is sufficiently close to Δ_u , there always exists $\eta \in [0, 1]$ and $\varepsilon > 0$ (both depending on the random draw Δ_t) such that $\Delta = \Delta_t$ is optimal and (80) holds. ■

Proof of Proposition 6. Parts (i)–(iii) are proved in the main body of the text. Part (iv) comes down to noticing that

$$\text{cov}(e^z, z) > 0 \quad (93)$$

for any random variable z — in particular, for $z = \omega(\Delta)$. The second statement of (iv) follows from the first and Jensen’s inequality applied to the convex function ω . ■

C An interpretation of participation costs

Throughout the paper we maintain the assumption that participation in “distant” markets incurs participation costs. In this appendix we discuss how these costs could arise as information-acquisition costs that permit an investor to avoid the lower net returns earned by an investor unfamiliar with the asset class. Indeed, we wish to re-emphasize that we do construe the notion of distance broadly, as a stand-in for the level of familiarity of investors in one location with all aspects of the financial environment in another.

We start by summarizing the set-up in Gârleanu et al. (2013), on which the interpretation is based. While we could simply refer the reader to this paper, we take this route in the interest of staying self-contained and keeping the effort required of the interested reader to a minimum.

Just as in the present model, investors are aligned on a circle. In each location of the circle, investors are of two types: a) “common” investors, who are endowed with local “regular” stocks that pay an exogenous, location-specific dividend, with the same properties as in the present paper; and b) “inside” investors, endowed with “fraudulent” firms in that location. Fraudulent firms produce lower dividends than regular firms — for simplicity it is assumed that they produce a dividend of zero. However, if insiders choose to, they can manipulate the earnings of their firms — for instance by diverting their personal funds into the company to boost the firm’s dividends. This assumption ensures that the insider always makes a trading profit by trading in her firm, irrespective of whether she buys or sells stock in her firm.²⁴

Similar to the present paper, investors see an incentive to invest in distant locations for diversification reasons. However, when investing in a location other than her own, an incompletely informed investor does not know which stocks are regular and which are fraudulent. Investors receive a signal about the type of every firm in every location. The quality of this signal deteriorates with the distance separating the investor from the firm. Hence, even though diversification benefits increase with distance, so does the informational disadvantage.

Gârleanu et al. (2013) shows that there exists an equilibrium in which insiders strategically manipulate the prices of the stocks they are endowed with, so that the prices of all risky securities in a given location are equal. The ability of insiders to manipulate the earnings of their firms implies that no trade with such an insider can be profitable for any other investor. This deters local short sellers from shorting the stock of the fraudulent firms.²⁵ Since prices are the same for fraudulent and regular securities, investors use only their private information in weeding out potential insiders and avoiding trading with them.

Because the ability to distinguish fraudulent from regular securities declines with distance, investors prefer to invest close to their own location and forego diversification opportunities. Indeed, in a special case where investors receive a (perfectly informative) signal for a subset of locations, and an uninformative signal for the rest, they choose to invest only in locations where they are informed, similar to the present paper.

Gârleanu et al. (2013) assumes that investors are simply endowed with their signals and doesn’t consider the possibility of incurring costs to expand the set of informative signals (and by implication the participation interval on which the investor chooses to invest.) By contrast, the focus of the present paper is to study investors’ incentives to to expand, by paying appropriate costs, the set of locations in which they can participate. Indeed, the endogeneity of the participation decision is at the core of many results in this paper, since it is responsible for the emergence of non-convexities, the diversity of financial strategies, the emergence of endogenous leverage, the vulnerability of the market equilibrium to sudden reversals in participation, etc.

D An alternative formulation of the leverage constraint

In this section we elaborate further on the interaction between borrowing constraints and high price sensitivity to participation costs. Specifically, we introduce a “limited-liability” constraint that

²⁴Vila (1989) shows the possibility of profitable manipulation, when the payoffs of a security can be manipulated.

²⁵The model assumes that local investors only observe prices, and not the demands expressed by “distant” investors for local fraudulent firms. Hence, if they sold short they would not know whether they were trading against less informed investors or the insider.

places an endogenous bound on borrowing, and show that it plays a similar role to constraint (22) in the text. In particular, we recover the conclusions of Section 5.

Before formalizing and analyzing the constraint, we provide a new dividend structure. An important novel feature of this structure is that all dividends are non-negative, so that the notion of limited liability is economically meaningful. Specifically, let Γ_j be a Gamma process on $[0, 1)$, so that for $u > s$ we have

$$\Gamma_u - \Gamma_s \sim \Gamma(k(u - s); \nu).$$

Extending $d\Gamma$ to the entire real line as before — that is, via $d\Gamma_s = d\Gamma_{s \bmod 1}$ — we define

$$D_j = \mu + \int_{j-\frac{1}{2}}^{j+\frac{1}{2}} w_{s-j} d\Gamma_s \quad (94)$$

for some $\mu \geq 0$ and weights $w_i \geq 0$ periodic with period 1 and symmetric around 0. In the interest of concreteness, in our numerical illustration below we define $w_i = 1$ if $i \in [-\frac{1}{4}, \frac{1}{4}]$ and $w_i = 0$ otherwise. Conveniently, for this choice of w , D_j and $D_{j+\frac{1}{2}}$ are independent.

More important, specification (94) generally implies that dividends are positive and the joint distribution of the dividends in any n locations depends exclusively on the distances on the circle between the locations.

An agent located in location i maximizes utility over end-of-period wealth $W_{1,i}$ net of participation costs, that is, she maximizes

$$-\frac{1}{\gamma} E_0 e^{-\gamma(W_{1,i} - F_i)},$$

where F_i refers to the participation costs incurred by the agent, depending on her participation choices. For the participation costs we adopt the same structure as in Section 5. Specifically we assume that by paying a cost κ , an investor can participate not only in her location but also in the location diametrically “opposite” hers on the circle. Otherwise the investor can only invest in the risky asset in her own location. Proceeding as in Section 5, the indifference of agent i between investing exclusively in location i and incurring the cost κ to participate also in location $i + \frac{1}{2}$ means

$$\max_{1-w_2^f} -E e^{-\gamma\left(\frac{1}{2}(1-w_2^f) \sum_{j=i, i+\frac{1}{2}} (D_j - P) - \kappa\right)} = \max_{1-w_1^f} -E e^{-\gamma\left((1-w_1^f)(D_i - P)\right)}, \quad (95)$$

where we have used the definition of an agent’s objective and her budget constraint. Note that $1 - w_2^f$, respectively $1 - w_1^f$, is the leverage choice of an agent who decides to invest across the two locations, respectively only in her own location.

In the absence of any constraint on leverage, the implicit function theorem applied to equation (95) yields

$$\frac{dP}{d\kappa} = -\frac{1}{w_1^f - w_2^f} < 0, \quad (96)$$

exactly as in Section 5.²⁶

Now suppose that, due to the no-recourse nature of lending contracts, borrowing is restricted so

²⁶It is possible to show that there exist values of κ for which only asymmetric equilibria exist.

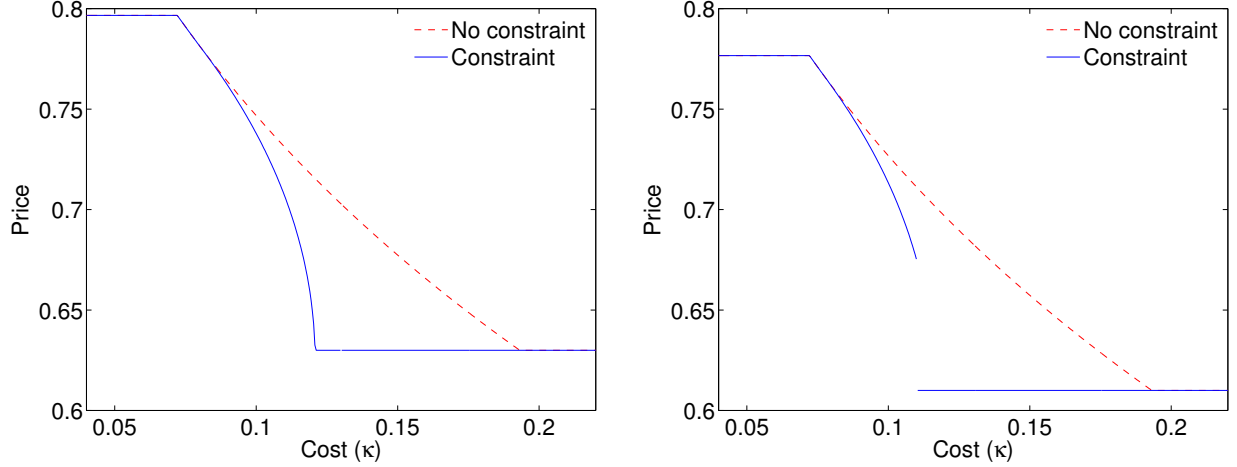


Figure 13: The figure illustrates the higher sensitivity of the price to the diversification cost in the presence of the constraint. In the left panel, the price decreases continuously to the value obtaining with no diversification. In the right panel, the price jumps to this value when $\kappa = \mu$. In either case, the slope of the solid line (the price in the presence of a leverage constraint) exhibits a steeper decline than the dotted line (the price in the absence of a leverage constraint). The common parameters used here are $k = 20$, $v = 10$, and $\gamma = 10$; in the left panel $\mu = 0.13$, while in the right one $\mu = 0.11$.

as to ensure that there is no default in equilibrium.²⁷ Thus, borrowing is subject to the constraint

$$X_2^S D^{\min} + (X_2^B - \kappa) \geq 0, \quad (97)$$

where D^{\min} is the smallest possible dividend in period 1, X_2^S is the number of shares chosen by investor a , and $-X_2^B$ is the amount borrowed by the investor. Using the time-zero budget constraint and noting that $D^{\min} = \mu$ and $X_2^S - 1 = -w_2^f$, equation (97) becomes

$$-w_2^f(P - \mu) \leq \mu - \kappa. \quad (98)$$

Equation (98) is key to the assertion that modeling the constraint in the way we do here induces small changes in κ to translate into larger drops in the price than would obtain absent the constraint. Repeating the steps in Section 5 we get

$$\frac{dP}{d\kappa} = -\frac{1 + \lambda}{w_2^f - (1 + \lambda)w_1^f} < -\frac{1}{w_2^f - w_1^f} < 0. \quad (99)$$

The intuition for the increased price sensitivity, captured by (99), is quite immediate: An increase in κ not only requires the price to decrease in order to avoid a decrease in V_2 relative to V_1 (the effect behind equation (96)); it also pushes the price down to counteract the direct effect

²⁷Richer contracts, through which the borrower and lender share some risk, can be envisaged. Note, however, that such a contract would allow an agent (partial) diversification across locations at zero cost, and thus run counter to the central friction of the paper.

of tightening of the constraint (decreasing the right-hand-side of (98)).

Depending on the parameters, the price may even drop discontinuously to the value obtaining in the no-diversification equilibrium. The point is made most starkly in the case $\kappa = 0$ and $\mu = 0$. At these values there is diversification, and no leverage. Any increase in κ , on the other hand, drives the price discontinuously down to the no-diversification value. More generally, it can happen that, as κ approaches μ , enough agents continue to diversify — even if their leverage is virtually zero — for the price to be above the no-diversification value that obtains when $\kappa > \mu$. However, once κ exceeds μ the price drops discontinuously, as the right panel of Figure 13 illustrates.

To conclude, even if one modeled borrowing limitations as resulting from a no-default requirement, the price function is steeper in κ than when the constraint is absent, and can even be discontinuous.